

# Fatigue Damage Prognosis in FRP Composites by Combining Multi-Scale Degradation Fault Modes in an Uncertainty Bayesian Framework

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## ABSTRACT

In this work, a framework for the estimation of the fatigue damage propagation in CFRP composites is proposed. Macro-scale phenomena such as stiffness and strength degradation are predicted by connecting micro-scale and macro-scale damage models in a Bayesian filtering framework that also allows incorporating uncertainties in the prediction. The approach is demonstrated on data collected from a run-to-failure tension-tension fatigue experiment measuring the evolution of fatigue damage in CRFP cross-ply laminates. Results are presented for the prediction of expected end of life for a given panel with the associated uncertainty estimates.

## INTRODUCTION

Fatigue in composite materials is a complex multi-scale damage cumulative process, and generally perceived as macro-scale reduction in stiffness and strength as structures age. Several fatigue models for both micro and macro scale phenomena have been proposed in literature that work reasonably well under fixed loading and material conditions [1], however, it is important to connect micro-scale damage to macro-scale properties to obtain reliable predictions in real engineering applications. Furthermore, there is much physical and modeling uncertainty that is not usually taken into account in such approaches. Hence an approach based on continuous assessment of structure health based on Bayesian state estimation methods is proposed that is better suited to (1) deal with different sources of uncertainty, (2) update model parameters as the system evolves, (3) use updated models to predict damage degradation into the future, which can be ultimately used to improve safety and reduce maintenance costs [2]. Other Bayesian approaches for fatigue modeling are mostly focused on making assumptions over the random variables [3] or over the relations between model and data [4]. Recently, there has been a growing interest for Bayesian methods in fatigue damage prognosis [5, 6] although mostly focused on crack propagation in metals. The

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application of a probabilistic prognostic model-based framework for predicting degradation in composite materials is still very limited in the literature.

However, the ability to deal with uncertainties from models and data can be the biggest advantage of Bayesian methods since the existence of uncertainty in composite materials is an undeniable fact.

In this work, a novel damage prognostics framework for FRP composites under fatigue loading is proposed. The key contribution is the inclusion of micro-scale damage evolution models acting as state transition equation (Figure 1), that are hierarchically connected to a macro-scale stiffness reduction model into a Bayesian filtering algorithm [7] that sequentially updates both damage states and model parameters as time evolves. Through stochastic embedding, these deterministic models are converted to probabilistic models by introducing a modeling error term. This modeling error term is controlled by a probability density function whose parameters are sequentially estimated in addition to the rest of model parameters.

The proposed methodology is implemented and demonstrated using experimental NDE damage data for micro-crack density and stiffness reduction from extensive tension-tension fatigue experiments performed over several symmetric cross-ply CFRP laminates [14].

## METHODOLOGY

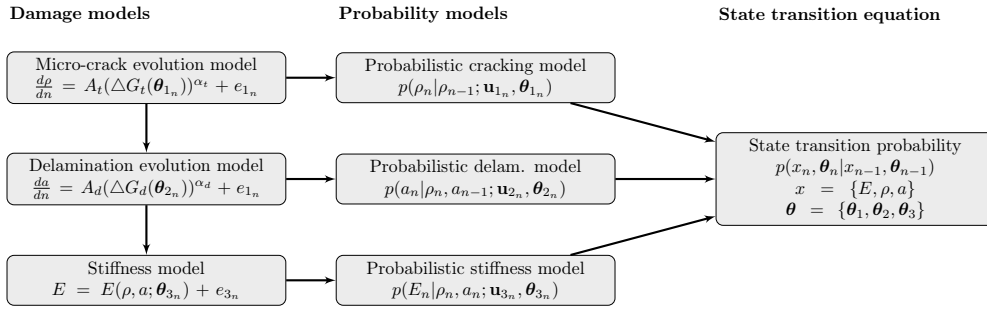


Figure 1: Deriving a probability transition kernel from deterministic multi-scale damage models for Bayesian filtering and prognostics

### Damage growth models

A Fracture Mechanics approach based on a modified Paris law is adopted to model the rate of change of internal damage per cycle. Several authors [8] have adopted a modified Paris law to analyze the rate of damage growth using the range of energy release rate instead of the range in stress intensity factor. The energy release rate  $\Delta G_t$  was calculated using a variational stress analysis approach. Other possible approaches can be the shear-lag analysis or the COD approach [8]. In this paper, the shear-lag approach to obtain the energy release rate is adopted, which is simpler and well-suited for symmetric cross-ply laminates<sup>2</sup>:

$$G_t = \frac{E_x^{(90)} \left( t_S E_x^{(S)} + t_{90} E_x^{(90)} \right)}{2 t_S E_x^{(S)} \xi} \left( \epsilon_0 + \epsilon_2^T \right)^2 \left( 2 \tanh \xi \rho t_{90} - \tanh 2 \xi \rho t_{90} \right) \quad (1)$$

<sup>2</sup>See last page for basic notation and relations

where

$$\xi = \sqrt{\frac{G}{d_0} \left( \frac{1}{E_x^{(S)} t_S} + \frac{1}{E_x^{(90)} t_{90}} \right)} \quad (2)$$

Then the modified Paris' Law for the propagation of matrix cracks is formulated as:

$$\frac{d\rho}{dn} = A_t (\Delta G_t)^{\alpha_t} \quad (3)$$

where  $A_t$  and  $\alpha_t$  are fitting parameters.  $\Delta(G)$  is the increment of energy release evaluated for the maximum and minimum stress in the cyclic load series:  $\Delta(G) = G_{|\sigma_{max}} - G_{|\sigma_{min}}$ . There is no closed-form solution for this differential equation, therefore we approximate the derivative by finite differences as:

$$\frac{\Delta\rho}{\Delta n} = \frac{\rho_n - \rho_{n-1}}{1} = A_t (\Delta G_t(\rho_{n-1}))^{\alpha_t} \quad (4)$$

hence

$$\rho_n = \rho_{n-1} + A_t (\Delta G_t(\rho_{n-1}))^{\alpha_t} \quad (5)$$

For macro-scale degradation, stiffness loss is preferred over residual strength given that (1) it can be measured non destructively, and (2) it exhibits much less statistical scatter than the strength [1]. For the case of cross-ply CFRP laminates, some authors [9] have shown that local delaminations affect relatively insignificantly to the global stiffness reduction when the stacking sequence  $[0_n/90_m/0_n]$  is such that  $m/n \leq 4$ , which is the case for our laminate. Therefore, the hypothesis that only matrix cracks are the dominant critical damage mode is adopted herein.

equation 6 shows the expression for relative stiffness reduction using a shear-lag based fracture mechanics model [10]:

$$\frac{E_{\text{eff}}}{E_0} = \frac{1}{1 + a\rho R(\bar{l})}, \quad \text{with } R(\bar{l}) = \frac{2}{\xi} \tanh(\xi \bar{l}) \quad (6)$$

Here,  $\xi$  is defined in equation 2. The term  $a$  is a known function dependent on elastic properties and geometry of the sub-laminate and  $90^\circ$  layer, defined as  $a = \frac{E_x^{(S)} t_{90}}{E_x^{(90)} t_S}$  [8, 10]. Hereinafter, we refer to the relation  $\frac{E_{\text{eff}}}{E_0}$  as  $D$ .

### Stochastic embedding

Any deterministic model of a system (e.g., a finite-element model, state-space model, or an ARMAX model) defining a relationship  $\{\mathbf{u}, \boldsymbol{\theta}\} \rightarrow \mathbf{g}(\mathbf{u}, \boldsymbol{\theta})$  between the model input  $\mathbf{u}$  and the model output  $\mathbf{g}$ , given a set of uncertain parameters  $\boldsymbol{\theta}$ , can be used to construct a probabilistic class by *stochastic embedding* [11]. This can be done by adding a model-error term  $\mathbf{v}$  to represent the difference between the real system output  $\mathbf{x}$  and the model output  $\mathbf{g}(\mathbf{u}, \boldsymbol{\theta})$ , as shown in equation (7a). If  $\mathbf{y}$  are the measurements of the system output  $\mathbf{x}$ , then one can extend the stochastic embedding to account for the measurement error  $\mathbf{w}$  as shown in equation (7b):

$$\mathbf{x} = \mathbf{g}(\mathbf{u}, \boldsymbol{\theta}) + \mathbf{v} \quad (7a)$$

$$\mathbf{y} = \mathbf{x} + \mathbf{w} \quad (7b)$$

Applying the stochastic embedding to the above damage models yields the following discrete-time state-space model:

$$\rho_n = \rho_{n-1} + g_1(\rho_{n-1}, \mathbf{u}_{1n}, \boldsymbol{\theta}_1) + v_{1n} \quad (8a)$$

$$D_n = g_2(\rho_n, \mathbf{u}_{2n}, \boldsymbol{\theta}_2) + v_{2n} \quad (8b)$$

$$\hat{\rho}_n = \rho_n + w_{1n} \quad (8c)$$

$$\hat{D}_n = D_n + w_{2n} \quad (8d)$$

where the error terms  $v_{j_n}$  and  $w_{j_n}$  are defined<sup>3</sup> as zero mean gaussians,  $\mathcal{N}(0, \sigma_{v_{j_n}})$  and  $\mathcal{N}(0, \sigma_{w_{j_n}})$  respectively. It follows that the probabilistic expressions for the state transition equations (8a, 8b) can be described as:

$$p(\rho_n | \rho_{n-1}, \mathbf{u}_{1n}, \boldsymbol{\theta}_1) = \mathcal{N}(\rho_{n-1} + g_1(\rho_{n-1}, \mathbf{u}_{1n}, \boldsymbol{\theta}_1), \sigma_{v_{1n}}) \quad (9a)$$

$$p(D_n | \rho_n, \mathbf{u}_{2n}, \boldsymbol{\theta}_2) = \mathcal{N}(g_2(\rho_n, \mathbf{u}_{2n}, \boldsymbol{\theta}_2), \sigma_{v_{2n}}) \quad (9b)$$

Similarly, the expressions for measurement equations (8c, 8d) are obtained as below:

$$p(\hat{\rho}_n | \rho_n) = \mathcal{N}(\rho_n, \sigma_{w_{1n}}) \quad (10a)$$

$$p(\hat{D}_n | D_n) = \mathcal{N}(D_n, \sigma_{w_{2n}}) \quad (10b)$$

### Filtering for Bayesian updating

The filtering framework consists on the sequential damage state assessment through Bayesian updating of the last state assessment as new data become available. Subsequently, the updated models at each step are run in a forward mode to predict estimate of *end of life* (EOL) or *remaining useful life* (RUL). In this model-based approach, the state assessment step includes estimation of the damage state  $x$  as well as estimation of model parameters,  $\boldsymbol{\theta}$ . Using *particle filters*, the joint state-parameter distribution  $p(x_n, \boldsymbol{\theta}_n | \hat{Y}_n)$  can be approximated by a set of  $N$  discrete weighted *particles*,  $\{(x_n^i, \boldsymbol{\theta}_n^i), \omega_n^i\}_{i=1}^N$ , as

$$p(x_n, \boldsymbol{\theta}_n | \hat{Y}_n) \approx \sum_{i=1}^N \omega_n^i \delta(x_n - x_n^i) \delta(\boldsymbol{\theta}_n - \boldsymbol{\theta}_n^i) \quad (11)$$

It must be noted that we simultaneously consider a micro-scale damage variable (transverse crack density,  $\rho$ ) and a macro-scale damage variable (stiffness loss,  $D$ ), such that each particle (state sample)  $x_n^i$  is composed as  $x_n^i = \{\rho_n^i, D_n^i\}$ . Hence, given sequences of both measurements,  $\hat{Y}_n = \{\hat{\rho}_n, \hat{D}_n\}$ , where  $\hat{\rho}_n = \{\hat{\rho}_0, \hat{\rho}_1, \dots, \hat{\rho}_n\}$  and  $\hat{D}_n = \{\hat{D}_0, \hat{D}_1, \dots, \hat{D}_n\}$ , equation 11 can be rewritten as<sup>4</sup>

$$p(\rho_n, D_n, \boldsymbol{\theta}_n | \hat{\rho}_n, \hat{D}_n) \approx \sum_{i=1}^N \omega_n^i \delta(\rho_n - \rho_n^i) \delta(D_n - D_n^i) \delta(\boldsymbol{\theta}_n - \boldsymbol{\theta}_n^i) \quad (12)$$

<sup>3</sup>A rational way to define a probability model for the error term could be to select it such that it produces the most uncertainty (largest Shannon entropy). The maximum-entropy PDF for an unbounded variable given its mean and variance is a Gaussian distribution.

<sup>4</sup>For simpler notation the conditioning on the model input sequences  $\mathbf{u}_{i_n}$ , that are supposed to be known in this problem, are dropped from the equation 12.

Applying Bayes' Theorem, the importance weights  $\omega_n^i$  can be updated as:

$$\omega_n^i \propto p(\hat{D}_n | D_n) p(\hat{\rho}_n | \rho_n) \omega_{n-1}^i \quad (13)$$

Here we assume that the system model is Markovian of order one and that the observations are conditionally independent of the state. We use the sampling importance resampling (SIR) particle filter, using systematic resampling. An artificial evolution approach [12] is also introduced to deal with the sequential updating of model parameters  $\theta_n$ . See below a pseudocode for this algorithm called Algorithm 1.

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**Algorithm 1** Particle Filter

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- 1: At  $n = 0$
  - 2: Generate  $\{\theta_0^i, x_0^i\}_{i=1}^N$ , sampling from the priors  $\pi_\theta(\cdot)$  and  $\pi_x(\cdot)$  respectively.
  - 3: Assign the initial weights:  $\{\omega_0^i = 1/N\}_{i=1}^N$
  - 4: At  $n \geq 1$
  - 5: **for**  $i = 1 \rightarrow N$  **do**
  - 6: Simulate from state equations:  $\theta_n^i \sim p(\cdot | \theta_{n-1}^i)$ ;  $\rho_n^i \sim p(\cdot | \rho_{n-1}^i, \theta_n^i)$ ;  $D_n^i \sim p(\cdot | \rho_n^i, \theta_n^i)$
  - 7: Update weights:  $\omega_n^i \propto p(\hat{D}_n | D_n^i) p(\hat{\rho}_n | \rho_n^i) \omega_{n-1}^i$
  - 8: **end for**
  - 9: **for**  $i = 1 \rightarrow N$  **do**
  - 10: Normalize  $\omega_n^i \leftarrow \omega_n^i / \sum_{i=1}^N \omega_n^i$
  - 11: **end for**
  - 12:  $\{(\rho_n^i, D_n^i, \theta_n^i), \omega_n^i\}_{i=1}^N \leftarrow \text{Resample}\{(\rho_n^i, D_n^i, \theta_n^i), \omega_n^i\}_{i=1}^N$
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### Damage prognostics

For predicting remaining useful life of a composite structure we are interested in predicting the time when the damage grows beyond a predefined acceptable threshold [13]. Using the most current knowledge of the system state at cycle  $n$ , estimated by equation (12), the goal now is to estimate the EOL, as probability:  $p(EOL_n | \hat{Y}_n)$ . The damage space itself may be defined by means of a set of thresholds  $\mathbf{C} = \{C_1, \dots, C_c\}$  on more than one critical parameters. In such cases, these thresholds can be combined into a *threshold function*  $T_{EOL} : T_{EOL}(x, \theta, \mathbf{C})$  that maps a given point in the joint state-parameter space to the Boolean domain  $\{0, 1\}$ . For instance, when a given particle  $i$  starting from cycle  $n$  performs a random walk and hits any of the thresholds  $\mathbf{C}$ , then  $T_{EOL}^i = 1$ , otherwise  $T_{EOL}^i = 0$ . The time  $n_T \geq n$  at which that happens defines the EOL for that particle. Mathematically:

$$EOL_n^i = \inf\{n_T \in \mathbb{N} : n_T \geq n \wedge T_{EOL}^i(x_{n_T}^i, \theta_{n_T}^i, \mathbf{C}_{n_T}) = 1\} \quad (14)$$

Using the updated weights at the starting time, a probabilistic estimation of the EOL is given as:

$$p(EOL_n | \hat{Y}_n) \approx \sum_{i=1}^N \omega_n^i \delta(EOL_n - EOL_n^i) \quad (15)$$

An algorithmic description of the prognostic procedure is provided as Algorithm 2.

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**Algorithm 2** EOL prediction

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1: Inputs:  $\{(\rho_n^i, D_n^i, \theta_n^i), \omega_n^i\}_{i=1}^N, \mathbf{C}_n = \{C_{1n}, \dots, C_{cn}\}$ 
2: Output:  $\{EOL_n^i, \omega_n^i\}_{i=1}^N$ 
3: for  $i = 1 \rightarrow N$  do
4:   Calculate:  $T_{EOL}^i(\rho_n^i, D_n^i, \theta_n^i, \mathbf{C}_n)$ 
5:   while  $T_{EOL}^i = 0$  do
6:     Simulate:  $\theta_{n+1}^i \sim p(\cdot | \theta_n^i); \rho_{n+1}^i \sim p(\cdot | \rho_n^i, \theta_{n+1}^i); D_{n+1}^i \sim p(\cdot | \rho_{n+1}^i, \theta_{n+1}^i)$ 
7:      $n \leftarrow n + 1$ 
8:      $(\rho_n^i, D_n^i, \theta_n^i) \leftarrow (\rho_{n+1}^i, D_{n+1}^i, \theta_{n+1}^i)$ 
9:   end while
10:   $EOL_n^i \leftarrow n$ 
11: end for
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**RESULTS AND DISCUSSION**

The proposed framework was applied to fatigue cycling data for cross-ply graphite-epoxy laminates. Torayca T700G uni-directional carbon-prepreg material was used for 15.24 [cm] x 25.4 [cm] coupons with dogbone geometry. The tests, as reported in [14], were conducted under load-controlled tension-tension fatigue loadings with a frequency of  $f = 5$  [Hz], a maximum stress of 80% of their ultimate stress, and a stress ratio  $R = 0.14$ . Laminate properties are summarized in Table 1. Lamb waves signals were periodically recorded using a PZT sensor network to estimate internal microcrack density. The mapping between PZT raw data and microcrack density was done following the methodology proposed in [15]. In addition, macro-scale damage measurements were taken using strain gauges at periodic intervals interspersed between fatigue cycling experiment. Results for sequential damage state estimation and prognostics are presented in Figure 2. To compute EOL, a set of damage thresholds  $\mathbf{C} = \{\rho_{max} = 0.4, D_{max} = 0.88\}$  was chosen. Figure 2a shows comparison between the crack density as estimated by the filtering algorithm and the crack density estimated from PZT sensors. Similarly, Figure 2b shows a good agreement between stiffness reduction as measured using strain gauge data and estimated by the particle filter. Every time new data arrive, damage is estimated and the updated model is further used to propagate damage into future to compute RUL, calculated as  $RUL_n = EOL_n - n$ . These predictions are plotted against time in Figure 2c. The two shaded cones of accuracy at 10% and 20% of true RUL help evaluate prediction accuracy and precision. Prediction precision clearly improves with time, however, accuracy seems to depart from true RUL at later stages, which indicates that the model and its variance structure do not fully capture the damage dynamics towards the end. Such behaviors have been reported earlier in [13] and may require further investigations to evaluate the tradeoffs between model fidelity and accuracy requirements from an application perspective. Finally, Figure 2d shows how some model parameters evolve through time as they get updated. Similar results were obtained when applied to other test coupon data, not shown here.

Long. Modulus $E_x$ [GPa]	Trans. Modulus $E_y$ [GPa]	In-plane Poisson $\nu_{xy}$	Out-of-plane Poisson $\nu_{yz}$	Shear modulus $E_s$ [GPa]	Thickness $t$ [mm]
137.5	8.4	0.309	0.5	6.2	0.132

Table 1: Ply properties used in the calculations.

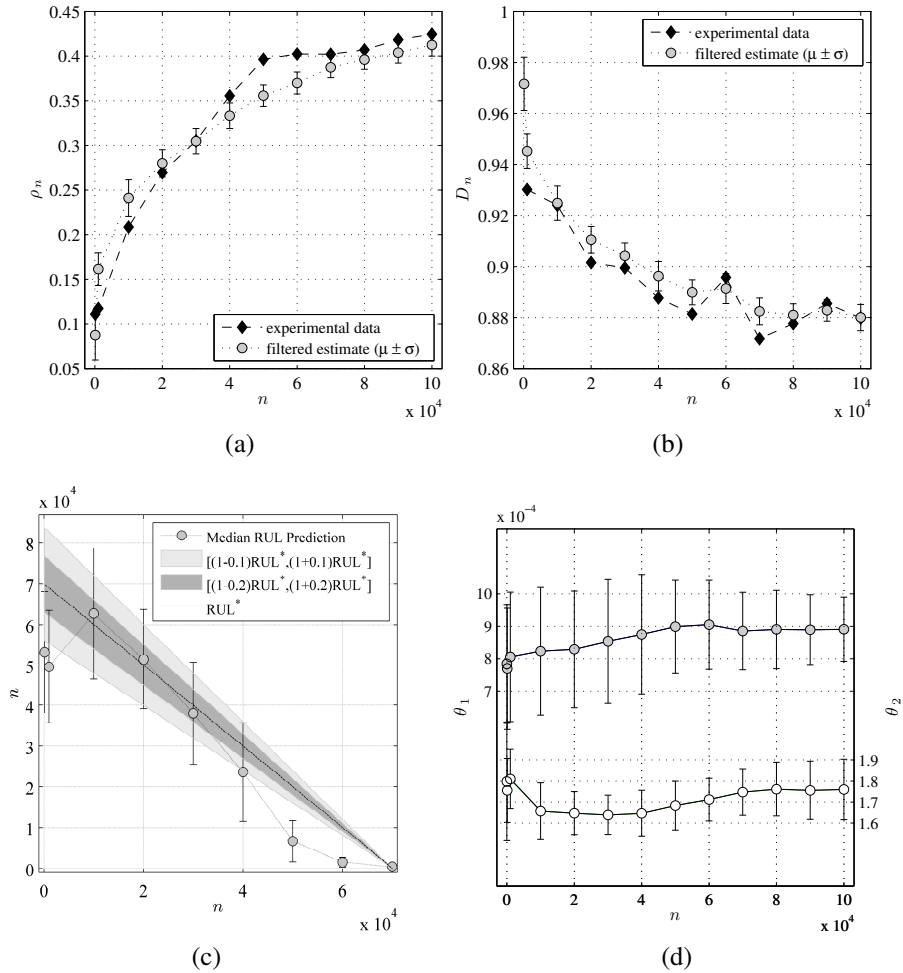


Figure 2: Sequential estate estimation of (a) microcrack density, (b) stiffness loss, (c) RUL prediction and (d) Evolution of Paris' Law parameters  $\{A_t, \alpha_t\}$

## CONCLUSIONS AND FUTURE WORK

A novel model-based filtering framework is proposed to sequentially update and predict the damage state, model parameters, and remaining useful life (RUL) of composites and estimate the uncertainty associated with these predictions. This is done by fusing the experimental information and models available at different levels of granularity by means of the Bayes' Theorem. Scope of future work includes (1) introducing delamination models and data into the proposed prognostics framework, (2) establishing energy-based thresholds to compute EOL for microcrack density and delamination growth, and (3) designing a robust filtering approach by fusing probabilistic information from a pool of plausible models.

## Basic notation

$h$	Laminate half-thickness	$\epsilon_0$	Applied strain
$t_S$	$[0_n]$ Sub-laminate thickness	$G$	Ply shear modulus
$t_{90}$	$[90_m]$ Sub-laminate half-thickness	$E_0$	Undamaged laminate Young's modulus
$l$	Half-distance between 2 cracks	$E_x^{(90)}$	Undamaged $[90_m]$ ply Young's modulus
$d_0$	Resin-rich thickness	$E_x^{(S)}$	Undamaged $[0_n]$ ply Young's modulus
$\bar{l} = \frac{l}{t_{90}}$	Dimensionless half spacing between cracks	$E_{\text{eff}}$	Damaged laminate Young's modulus

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