Chapter 2
Advances in Uncertainty Representation and Management for Particle Filtering Applied to Prognostics

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Abstract Particle filters (PF) have been established as the de facto state of the art in failure prognosis. They combine advantages of the rigors of Bayesian estimation to nonlinear prediction while also providing uncertainty estimates with a given solution. Within the context of particle filters, this paper introduces several novel methods for uncertainty representations and uncertainty management. The prediction uncertainty is modeled via a rescaled Epanechnikov kernel and is assisted with resampling techniques and regularization algorithms. Uncertainty management is accomplished through parametric adjustments in a feedback correction loop of the state model and its noise distributions. The correction loop provides the mechanism to incorporate information that can improve solution accuracy and reduce uncertainty bounds. In addition, this approach results in reduction in computational burden. The scheme is illustrated with real vibration feature data from a fatigue-driven fault in a critical aircraft component.

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2.1 Introduction

Uncertainty management of prognostics holds the key for a successful penetration of prognostics as a key enabler to health management in industrial applications. While techniques to manage the uncertainty in the many factors contributing to current health state estimation – such as signal-to-noise ratio (SNR) on diagnostic features, optimal features with respect to detection statistics and ambiguity set minimization – have received a fair amount of attention due to the maturity of the diagnostics domain, uncertainty management for prognostics is an area which still awaits significant advances.

Those shortcomings non-withstanding, a number of approaches have been successfully suggested for uncertainty representation and management in prediction. In particular, probabilistic, soft computing methods and tools derived from evidential theory or Dempster–Shafer theory [1] have been explored for this purpose. Probabilistic methods are mathematically rigorous assuming that a statistically sufficient database is available to estimate the required distributions. Possibility theory (fuzzy logic) offers an alternative when scarce data and even incomplete or contradictory data are available. Dempster's rule of combination and such concepts from evidential theory as belief on plausibility (as upper and lower bounds of probability) based on mass function calculations can support uncertainty representation and management tasks. The authors in [2] introduced a Neural Network construct called Confidence Prediction Neural Network to represent uncertainty in the form of a confidence distribution while managing uncertainty via learning during the prediction process. The scheme employs Parzen windows as the kernel and the network is based on Specht's General Regression Neural Network [3]. Radial Basis Function Neural Nets (RBFNN), Probabilistic Neural Nets (PNN) and other similar constructs from the neural net and neuro-fuzzy arena have been deployed as candidates for uncertainty representation and management. For example, Leonard et al. [4] used an RBFNN to obtain confidence limits for a prognosticator. Probabilistic reliability analysis tools employing an inner-outer loop Bayesian update scheme [5, 6] have also been used to "tune" model hyperparameters given observations. However, the scalability of this rigorous approach for more than a few parameters is unproven and relies on the assumption that all distributions are unimodal.

This paper introduces a generic and systematic methodology to the uncertainty representation and management problem in failure prognosis by capitalizing on notions from Bayesian estimation theory and, specifically, particle filtering (PF) [7, 9–10] for long-term prognosis in non-linear dynamic systems with non-Gaussian noise, appropriate kernels to reduce the impact of model errors and feedback correction loops to improve the accuracy and precision of the remaining useful life estimates. Prediction uncertainty is modeled via rescaled Epanechnikov kernels, considering the current state pdf estimate as initial condition of stochastic dynamic models, and is assisted with regularization algorithms. Uncertainty management is accomplished through parametric adjustments in a feedback correction loop of the state model and its noise distributions. It is assumed that for a specific application domain, the sources of uncertainty have been identified, raw data are available (for example, vibration data, load profiles, etc.), key fault indicators or features are extracted on-line from such sensor data that are characteristic of the health of critical components/subsystems, and fault detection, isolation and identification routines exploit these features to classify with prescribed confidence and false alarm rates the presence of a fault. In a probabilistic fault diagnosis framework, these features are expressed as probability density functions and are used to initialize the prognostic algorithms.

2.2 Uncertainty Representation and Management in Long-Term Prediction: A Particle Filtering-Based Approach

2.2.1 Failure Prognosis and Uncertainty Representation

Nonlinear filtering is defined as the process of using noisy observation data to estimate at least the first two moments of a state vector governed by a dynamic nonlinear, non-Gaussian state-space model. From a Bayesian standpoint, a nonlinear filtering procedure intends to generate an estimate (of the posterior probability density function $p(x_t \mid y_{1:t})$ for the state, based on the set of received measurements. Particle Filtering (PF) is an algorithm that intends to solve this estimation problem by efficiently selecting a set of N particles $\{x^{(i)}\}_{i=1\ldots N}$ and weights $\{w_t^{(i)}\}_{i=1\ldots N}$, such that the state pdf may be approximated by [7]

$$\tilde{\pi}_t^N(x_t) = \sum_{i=1}^{N} w_t^{(i)} \delta(x_t - x_t^{(i)}). \tag{2.1}$$

Prognosis, and thus the generation of long-term prediction, is a problem that goes beyond the scope of filtering applications since it involves future time horizons. Hence, if PF-based algorithms are to be used, it is necessary to propose a procedure with the capability to project the current particle population in time in the absence of new observations [7].

Any adaptive prognosis scheme requires the existence of at least one feature providing a measure of the severity of the fault condition under analysis (fault dimension). If many features are available, they can always be combined to generate a single signal. In this sense, it is always possible to describe the evolution in time of the fault dimension through the nonlinear state equation:

$$\begin{cases} x_1(t+1) = x_1(t) + x_2(t) \cdot F(x(t), t, U) + \omega_1(t), \\ x_2(t+1) = x_2(t) + \omega_2(t), \end{cases}$$

$$y(t) = x_1(t) + \upsilon(t), \tag{2.2}$$

where $x_1(t)$ is a state representing the fault dimension under analysis, $x_2(t)$ is a state associated with an unknown model parameter, U are external inputs to the system (load profile, etc.), $F(x(t), t, U)$ is a general time-varying nonlinear function, $y(t)$ represents feature measurements, and $\omega_1(t)$, $\omega_2(t)$ and $\upsilon(t)$ are white noises (not necessarily Gaussian). The nonlinear function $F(x(t), t, U)$ may represent a model based on first principles, a neural network, or even a fuzzy system.

By using the aforementioned state equation to represent the evolution of the fault dimension in time, it is possible to generate long term predictions for the state pdf in a recursive manner using the current pdf estimate for the state [7]:

$$\begin{aligned}\tilde{p}(x_{t+p} \mid y_{1:t}) &= \int \tilde{p}(x_t \mid y_{1:t}) \prod_{j=t+1}^{t+p} p(x_j \mid x_{j-1})\, \mathrm{d}x_{t:t+p-1} \qquad (2.3)\\ &\approx \sum_{i=1}^{N} w_t^{(i)} \int \cdots \int p(x_{t+1} \mid x_t^{(i)}) \prod_{j=t+2}^{t+p} p(x_j \mid x_{j-1})\, \mathrm{d}x_{t+1:t+p-1}.\end{aligned}$$

The evaluation of these integrals, though, may be difficult and/or may require significant computational effort. This paper proposes a methodology to solve this problem using a PF algorithm to estimate the current state pdf (from feature measurements associated to the fault dimension) and a combination of rescaled kernel functions and resampling schemes to reconstruct the estimate of the state pdf for future time instants.

Consider, in this sense, a discrete approximation for the predicted state pdf

$$\hat{p}(x_{t+k} \mid \hat{x}_{1:t+k-1}) \approx \sum_{i=1}^{N} w_{t+k-1}^{(i)} K\big(x_{t+k} - E[x_{t+k}^{(i)} \mid \hat{x}_{t+k-1}^{(i)}]\big), \qquad (2.4)$$

where $K(\cdot)$ is a kernel density function, which may correspond to the process noise pdf, a Gaussian kernel or a rescaled version of the Epanechnikov kernel:

$$K_{\mathrm{opt}}(x) = \begin{cases} \dfrac{n_x + 2}{2c_{n_x}}\big(1 - \|x\|^2\big) & \text{if } \|x\| < 1,\\ 0 & \text{otherwise,}\end{cases} \qquad (2.5)$$

where c_{n_x} is the volume of the unit sphere in $\mathbb{R}^{n_x}$.

Furthermore, if the density is Gaussian with unit covariance matrix, the optimal bandwidth is given by

$$\begin{aligned} h_{\mathrm{opt}} &= A \cdot N^{-1/n_x+4},\\ A &= \Big(8c_{n_x}^{-1} \cdot (n_x + 4) \cdot \big(2\sqrt{\pi}\big)^{n_x}\Big)^{1/n_x+4}. \end{aligned} \qquad (2.6)$$

The Epanechnikov kernel is particularly recommended in the special case of equally weighted samples [8], and thus it is well suited for uncertainty representation in long term predictions where no future measurements are available for a weight update procedure.

Given $\{x_t^{(i)}\}_{i=1\ldots N}$ and $\{w_t^{(i)}\}_{i=1\ldots N}$ as initial conditions, it is possible to represent the uncertainty inherent to the predicted state pdf by performing an inverse transform resampling procedure for the particle population [7]. This method obtains a fixed number of samples for each future time instant, avoiding problems of excessive computational effort. In fact, after the resampling scheme is performed, the weights may be expressed as: $\{w_{t+k}^{(i)}\}_{i=1\ldots N} = N^{-1}$. Furthermore, if only Epanechnikov kernels are used, it is ensured that the representation of the uncertainty will be bounded.

To avoid loss of diversity among particles and minimize the effect of model errors in the long term predictions, an additional step inspired by the Regularized Particle Filter [7] is performed for $k > 1$. In this step, it is assumed that the state covariance matrix $\hat{S}_{t+k}$ is equal to the empirical covariance matrix of $\hat{x}_{t+k}$:

Long Term Predictions: Regularization of Predicted State PDF

- Apply modified inverse transform resampling procedure.
  For $i = 1, \ldots, N$, $w_{t+k}^{(i)} = N^{-1}$
- Calculate $\hat{S}_{t+k}$, the empirical covariance matrix of
  $\{E[x_{t+k}^{(i)} \mid |\hat{x}_{t+k-1}^{(i)}], w_{t+k}^{(i)}\}_{i=1}^{N}$
- Compute $\hat{D}_{t+k}$ such that $\hat{D}_{t+k}\hat{D}_{t+k}^{T} = \hat{S}_{t+k}$
- For $i = 1, \ldots, N$, draw $\varepsilon^i \sim K$, an Epanechnikov kernel and assign
  $\hat{x}_{t+k}^{(i)*} = \hat{x}_{t+k}^{(i)} + h_{t+k}^{\text{opt}}\hat{D}_{t+k}\varepsilon^i$

It must be noted that the proposed method for uncertainty representation allows considering information for on-line measurements to estimate the uncertainty that is present in the system at the moment that the long-term predictions are generated. Moreover, the use of kernel functions and resampling techniques (with a limited number of particles) naturally permits to represent uncertainty in future time instants, just as other approaches, but using less computational resources. Figure 2.1 shows a representation of the particle filtering-based uncertainty representation scheme.

After the completion of the algorithm, it is possible to isolate the particles that define the bounds for the predicted pdf at future time instants. The collection of all these particles results in minimum and maximum bounds for the predicted state in time. Moreover, these bounds intrinsically incorporate, measure, and represent model uncertainty (through the estimation of unknown parameters) and measurement noise (since the initial condition for long-term predictions corresponds to the output of the Particle Filtering procedure).

For the test case of a fatigue failure in a critical aircraft component discussed in the sequel, (treated in Section 2.4 of this paper), the uncertainty representation results are shown in Figure 2.2.

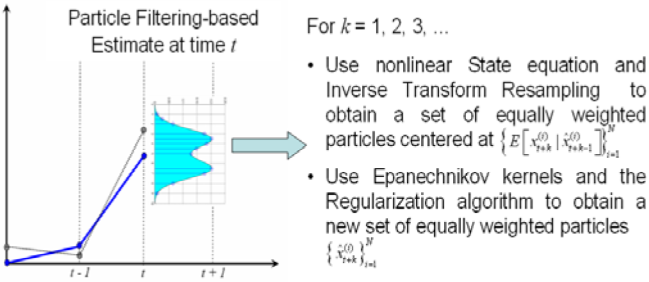

Fig. 2.1 Particle filtering-based uncertainty representation.

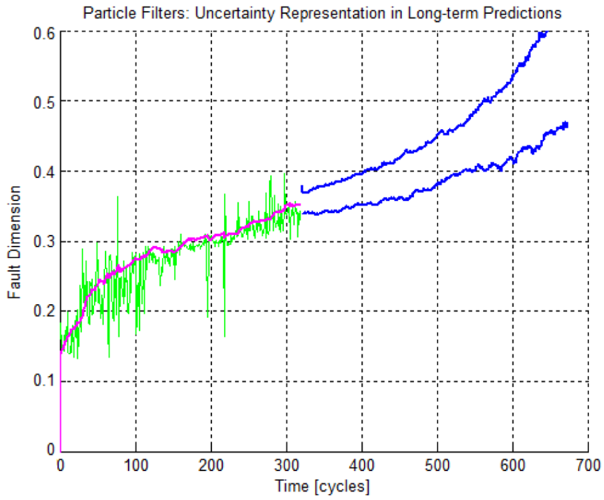

Fig. 2.2 Particle filtering-based uncertainty representation; results for the case of a fatigue fault progression in a critical aircraft component.

2.3 Uncertainty Management in Long-Term Predictions

The issue of uncertainty management, in a Particle Filtering-based prognosis framework, is basically related to a set of techniques aimed to improve the estimate at the current time instant, since both the expectation of the predicted trajectories for particles and bandwidth of Epanechnikov kernels depend on that pdf estimate.

In this sense, it is important to distinguish between two main types of adjustments that may be implemented to improve the current representation of uncertainty for future time instants:

- Adjustments in unknown parameters in the state equation.
- Adjustments in the parameters that define the noise pdf's embedded in the state equation. These parameters will be referred to as "hyperparameters".

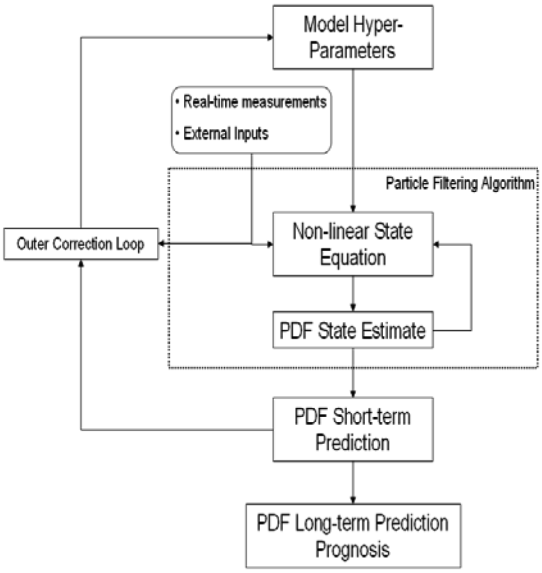

Fig. 2.3 Particle filtering-based uncertainty management system.

Accuracy of long-term predictions is directly related to the estimates of xt and the model hyper-parameters that affect E[xtlxt-1]. Precision in long-term predictions, on the other hand, is directly related to the hyper-parameters that describe the variance of the noise structures considered in the state equation. In this sense, any uncertainty management system for future time instants, within a Particle Filtering-based framework, should follow the general structure presented in Figure 2.3, where the performance of the algorithm is evaluated in terms of the short-term prediction error (which depends on the PF-based pdf estimate). Whenever the performance criteria for the short-term prediction error are not met, an *outer correction loop* directly modifies both model parameters and hyper-parameters.

By modifying the hyper-parameters via an *outer correction loop*, short-term predictions may be used to improve the efficiency of the particle filtering-based estimate, and thus the subsequent generation of long-term predictions. There is a large variety of *outer correction loops* that may be applied for this purpose. One of them, aimed to modify the variance of the noise term in the state equation, is proposed and analyzed in [7]. In this case, let $\omega_2(t)$ represent the model uncertainty. Then for the example detailed in the next section of the paper, the recommended parameters are:

$$\begin{cases} \text{var}\{\omega_2(t+1)\} = p \cdot \text{var}\{\omega_2(t)\}, \text{ if } \dfrac{\|\text{Pred\_error}(t)\|}{\|\text{Feature}(t)\|} < Th, \\ \text{var}\{\omega_2(t+1)\} = q \cdot \text{var}\{\omega_2(t)\}, \text{ if } \dfrac{\|\text{Pred\_error}(t)\|}{\|\text{Feature}(t)\|} > Th, \end{cases} \tag{2.7}$$

where Pred_error(t) is the short-term prediction error computed at time t, $\|\cdot\|$ is any well-defined norm (usually L_2-norm), $0 < p < 1$, $q > 1$, and $0 < Th < 1$ are scalars. In particular, $p \in [0.925, 0.975]$, $q \in [1.10, 1.20]$, and $Th = 0.1$. These values have been determined through exhaustive analysis of simulations considering scenarios with different combinations of values for the parameters p, q, and Th. The range for short-term predictions depends on the system under analysis, although a 5-step is recommended to ensure rapid adaptation of the scheme.

Outer correction loops may be also implemented using neural networks, fuzzy expert systems, PID controllers, among others. Additional correction loops include the modification of the number of particles used for 1-step or long-term prediction purposes and the reduction of the threshold for the use of the importance resampling algorithm.

2.4 An Illustrative Example

As an illustrative example, consider the case of propagating fatigue crack on a critical component in a rotorcraft transmission system. The objective in this seeded fault test is to analyze how a cyclic load profile affects the growth of an axial crack. Although the physics-based model for a system of these characteristics may be complex, it is possible to represent the growth of the crack (fault dimension) using the much simpler population-growth-based model [9, 10]:

$$\begin{cases} x_1(t+1) = x_1(t) + C \cdot x_2(t) \cdot (a - b \cdot t + t^2)^m + \omega_1(t), \\ x_2(t+1) = x_2(t) + \omega_2(t), \end{cases}$$
$$y(t) = x_1(t) + \upsilon(t), \tag{2.8}$$

where $x_1(t)$ is a state representing the fault dimension, $x_2(t)$ is a state associated with an unknown model parameter, $y(t)$ are vibration-based feature measurements, C and m are constants associated with the fatigue properties of the material. The constants a and b depend on the maximum load and duration of the load cycle (external input U). Given a set of vibration feature data such as described in [11], it is possible to use this model to obtain an approximate (and noisy) estimate of the crack length via the use of a PF-based algorithm [7]. Once the estimate of the state pdf is available, it can be used as initial condition of the model to generate long-term predictions, if the integrals in the aforementioned recursive expression are evaluated.

Figures 2.4 and 2.5 show the results for the application of (the regularization procedure to $\hat{p}(x_{t+1} \mid \hat{x}_{1:t})$, i.e. the predicted state pdf for time $t+1$, in the example case. First, the PF-based algorithm is used to obtain a pdf state estimate for the state model (2.8) at time $t = 320$. The analysis of feature data at that point indicates that the length of the crack is approximately 0.35 units. It is of interest to prognosticate the time instant when the fault dimension reaches 0.45 units, which is the expected

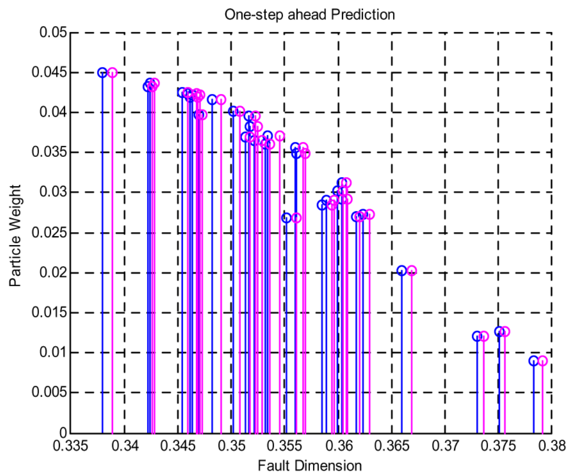

Fig. 2.4 One-step ahead prediction from a Particle Filtering standpoint. The blue samples reprsent the state pdf at time $t = 320$, while the magenta samples illustrate $\hat{p}(x_{321} \mid \hat{x}_{1:320})$.

value of the hazard zone of the component under consideration [7]. Figure 2.4 illustrates how the use of model (2.8) helps to propagate the particle population in time: $E[x_{t+1}^{(i)} \mid \hat{x}_t^{(i)}]$ is computed, and the particle weights are kept constant. As a result, the expected value of the crack length should increase in time.

The second step of the proposed algorithm is the regularization procedure; see Figure 2.5. This step modifies the whole particle population to improve the uncertainty representation for the predicted state pdf at time $t = 321$. As a result, a new population of equally weighted samples is obtained representing the probability density function at that particular time. The procedure may be repeated as needed until the expected value of the population reaches 0.45 units.

For the uncertainty management scheme, a 5-step prediction error has been used in the design of the *outer correction loop*. As expected, the longer the period used to calculate the prediction error, the larger the delay in the feedback loop. Several aspects must be considered in a proper selection of this parameter – as well as for p, q and Th in Equation (2.7) – including time constants of the system and the variability of model parameters.

The final implementation of the *correction loop* is shown in (2.9), while Figure 2.6 shows the results.

$$\begin{cases} \mathrm{var}\{\omega_2(t+1)\} = 0.95 \cdot \mathrm{var}\{\omega_2(t)\}, \text{ if } \dfrac{\|\text{Pred\_error}(t)\|}{\|y(t)\|} < 0.1, \\ \mathrm{var}\{\omega_2(t+1)\} = 1.20 \cdot \mathrm{var}\{\omega_2(t)\}, \text{ if } \dfrac{\|\text{Pred\_error}(t)\|}{\|y(t)\|} > 0.1, \end{cases} \tag{2.9}$$

Figure 2.6 clearly shows a period of time where the *outer correction loop* actually increments the variance of the noise profile $\omega_2(t)$ in the dynamic model (2.8);

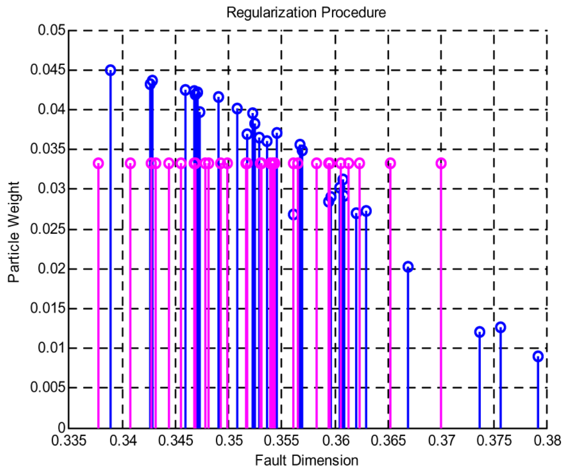

Fig. 2.5 Illustration of regularization procedure.

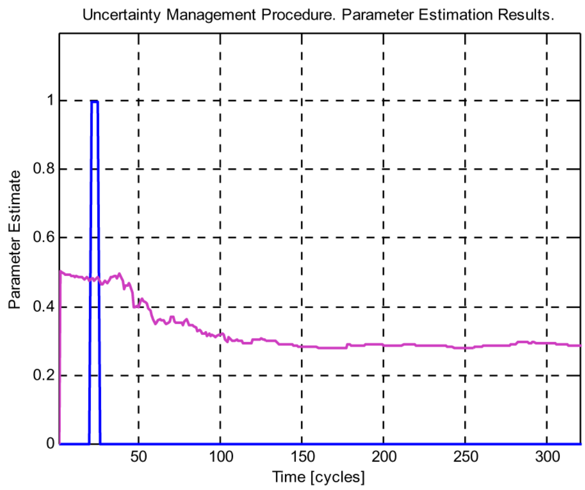

Fig. 2.6 Uncertainty management procedure; results of hyper-parameters update via an outer correction loop.

see blue lines in Figure 2.6. After this period, it is observed that the state estimate (magenta line) rapidly converges. After that condition is reached, the prediction error decreases considerably and therefore the variance of the noise used for the "artificial evolution" estimation method decreases exponentially.

Figure 2.7 depicts the results obtained when computing the pdf of the Remaining Useful Life (RUL) for the same test case under study, considering that the critical fault dimension corresponds to 0.45 units. A procedure to obtain the RUL pdf from the predicted path of the state pdf is detailed and discussed in [7]. Basically, the

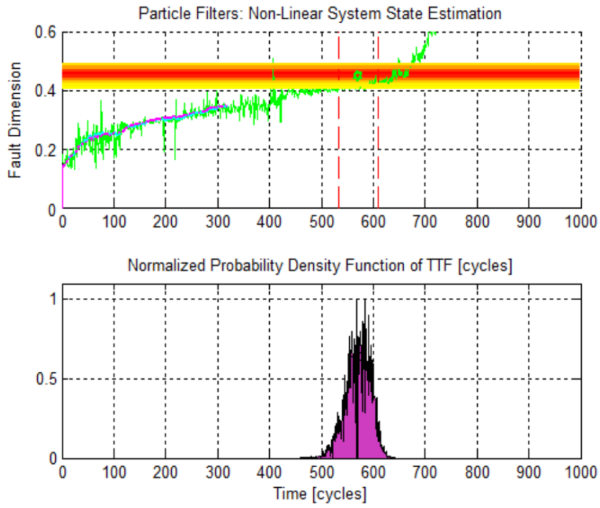

Fig. 2.7 Particle filtering-based uncertainty management system. Prognosis results for the case of axial crack in a gearbox plate.

RUL pdf is the probability of failure at future time instants. This probability can be obtained from long-term predictions, when the empirical knowledge about critical conditions for the system is included in the form of thresholds for main fault indicators, also referred to as the hazard zones. The hazard zone in this case is represented as an orange horizontal band around 0.45 units in Figure 2.7. Once the RUL pdf estimate is generated, it is possible to obtain any necessary statistics about the evolution of the fault in time, either in the form of expectations or 95% confidence intervals.

It is desired for the expected value of this RUL pdf (computed at time $t = 320$) to be close to the actual time-to-failure. In this sense, the efficacy of the approach can be evaluated using two performance metrics: accuracy and precision. Accuracy measures how close the RUL expectation is to the actual time-to-failure. Precision, on the other hand, indicates the variance associated to RUL estimates. Precise RUL estimates imply a pdf with small variance.

The performance of the proposed Particle filtering-based approach has been compared with another implementation that uses the Extended Kalman filter to generate an estimate of the current state pdf. Results indicate that the Particle filteringbased approach, in combination with the proposed *outer correction loop*, provides better results in terms of accuracy and precision of the RUL pdf estimates; see Figure 2.8. Moreover, both applications require a similar amount of time to perform all the calculations, given that the Particle filterbased prognosis algorithm does not necessitate an extremely large number of particles to achieve its objective [7].

More accurate and more precise pdf estimates provide the basis for timely corrective actions and thus, help to avoid catastrophic failure events during the operation of the unit/process under supervision. In this sense, the proposed methodology

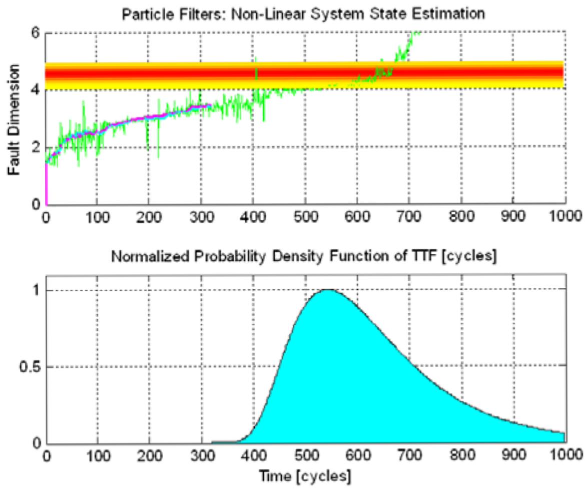

Fig. 2.8 Prognosis results for the case of axial crack in a gearbox plate, using an Extended Kalman filter-based approach.

has proven to be a valuable tool in Prognostics and Health Management (PHM) systems.

2.5 Conclusion

Uncertainty representation and management in failure prognosis present major challenges to the PHM user community. The stochastic nature of long-term predictions necessitates appropriate and effective means that can represent and manage uncertainty in almost real time. The on-platform utility of health and usage monitoring systems must be accompanied by robust prognostic algorithms if these technologies are to provide useful health information impacting safety of operation and cost of ownership. This paper proposes advances to uncertainty representation and management that will help in faster, more accurate, and more precise prognostics. Specifically, an architecture has been proposed that features feedback correction loops that in turn can reduce the impact of model errors and thus improve the accuracy and precision of the remaining useful life estimates. In addition, the use of appropriate kernels allows for an elegant and fast updating mechanism within the particle filter paradigm.

Results were obtained using data from a cracked gearbox plate. The performance (accuracy and precision) of the particle filter was superior when compared to a more traditional Extended Kalman Filter approach. Future work should explore the impact of injecting additional information into the correction loop such as domain expert information, maintenance information, etc. In addition, learning algorithms should be investigated that can automatically establish optimal or near-optimal values for parameters p, q, and Th and for the range for short-term prediction (here arbitrarily set to 5).

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