Adding New Physics and Numerics into the BATSRUS MHD Code

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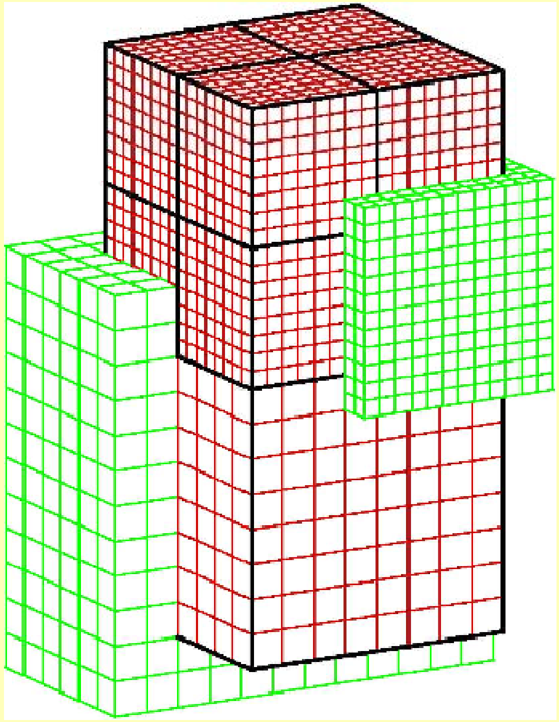
Outline

- BATS-R-US code
- Space Weather Modeling Framework
- Two fluid MHD and non-isotropic pressure
- Multi-ion MHD results
- Local time-stepping scheme
- Summary

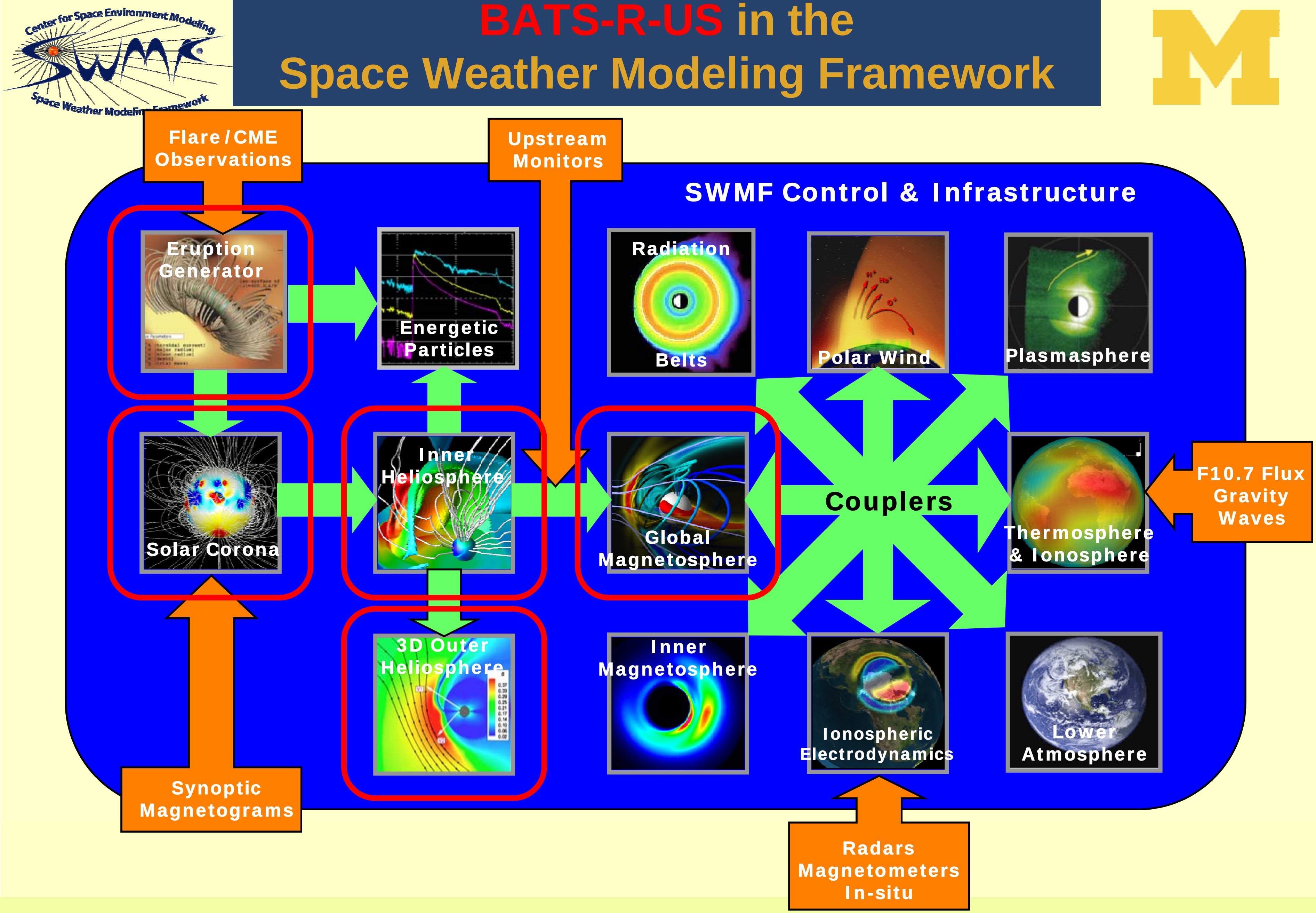
BATS-R-US

Block Adaptive Tree Solar-wind Roe Upwind Scheme



- **Physics**
  - Classical, semi-relativistic and Hall MHD
  - Multi-species, multi-fluid, anisotropic pressure
  - Radiation hydrodynamics with gray diffusion
  - Multi-material, non-ideal equation of state
  - Solar wind turbulence
- **Numerics**
  - Conservative finite-volume discretization
  - Parallel block-adaptive grid
  - Cartesian and generalized coordinates
  - Splitting the magnetic field into $B_0 + B_1$
  - Divergence B control: 8-wave, CT, projection, parabolic/hyperbolic
  - Shock-capturing TVD schemes: Rusanov, HLLE, AW, Roe, HLLD
  - Explicit, point-implicit, semi-implicit, fully implicit time stepping
- **Applications**
  - Sun, heliosphere, magnetosphere, unmagnetized planets, moons, comets…
- **100,000+ lines of Fortran 90 code with MPI parallelization**

BATS-R-US in the Space Weather Modeling Framework



The SWMF is freely available at http://csem.engin.umich.edu

Resistive Hall MHD with electrons and anisotropic ion pressure

Mass conservation: $$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0$$

Momentum: $$\frac{\partial \rho\mathbf{u}}{\partial t} + \nabla\cdot(\rho\mathbf{u}\mathbf{u} + P) = \mathbf{J}\times\mathbf{B}$$

$$\frac{\partial \rho\mathbf{u}}{\partial t} + \nabla\cdot(\rho\mathbf{u}\mathbf{u} + pI) = -en_e(\mathbf{E} + \mathbf{u}\times\mathbf{B})$$

$$P = (p_\perp + p_e)I + (p_\parallel - p_\perp)\mathbf{b}\mathbf{b} \qquad \mathbf{b} = \mathbf{B}/B$$

Induction: $$\frac{\partial \mathbf{B}}{\partial t} + \nabla\times\mathbf{E} = 0$$

Pressure: $$\frac{\partial p_\perp}{\partial t} + \nabla\cdot(p_\perp\mathbf{u}) = +\frac{2\eta e^2 n_e}{M_i}(p - p_e) - p_\perp\nabla\cdot\mathbf{u} + p_\perp\mathbf{b}\cdot(\nabla\mathbf{u})\cdot\mathbf{b}$$

$$\frac{\partial p}{\partial t} + \nabla\cdot(p\mathbf{u}) = (\gamma - 1)\left[-p\nabla\cdot\mathbf{u} + \eta\mathbf{J}^2 + \frac{3\eta e^2 n_e}{M_i}(p_e - p)\right]$$

$$\frac{\partial p_\parallel}{\partial t} + \nabla\cdot(p_\parallel\mathbf{u}) = +\frac{2\eta e^2 n_e}{M_i}(p - p_e) - 2p_\parallel\mathbf{b}\cdot(\nabla\mathbf{u})\cdot\mathbf{b}$$

Electron pressure:

$$\frac{\partial p_e}{\partial t} + \nabla\cdot(p_e\mathbf{u}_e) = (\gamma - 1)\left[(-p_e\nabla\cdot\mathbf{u}_e + \eta\mathbf{J}^2 + \frac{3\eta e^2 n_e}{M_i}(p - p_e) + \nabla\cdot(\kappa\mathbf{b}\mathbf{b}\cdot\nabla T_e)\right]$$

Electron velocity: $$\mathbf{u}_e = \mathbf{u} - \frac{\mathbf{J}}{en_e}$$

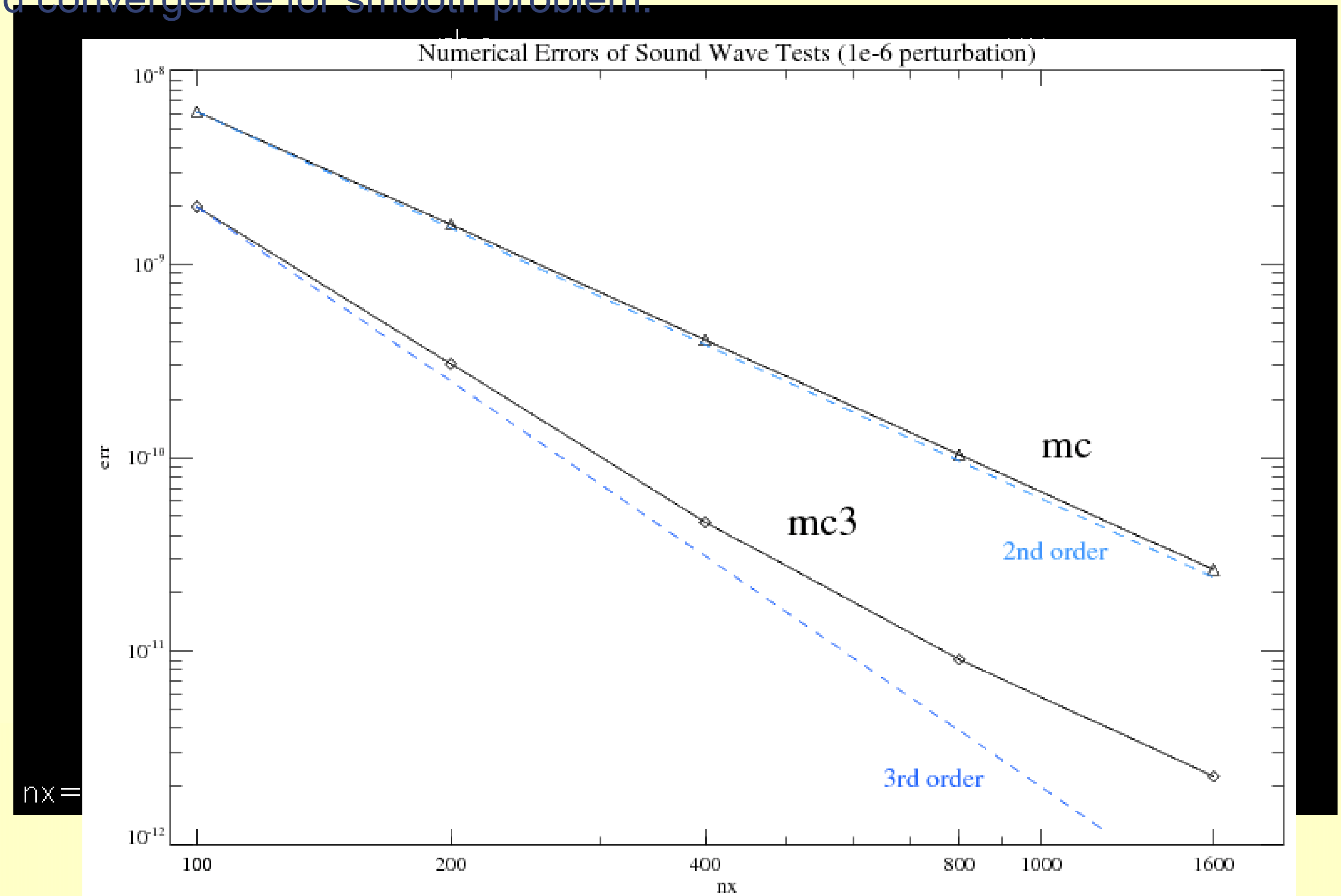
Electric field: $$\mathbf{E} = -\mathbf{u}\times\mathbf{B} + \eta\mathbf{J} + \frac{\mathbf{J}\times\mathbf{B}}{en_e} - \frac{\nabla p_e}{en_e} - \frac{\nabla\cdot[\mathbf{b}\mathbf{b}(p_\parallel - p_\perp)]}{en_e}$$

Current: $$\mathbf{J} = \nabla\times\mathbf{B}$$

Tests for anisotropic pressure

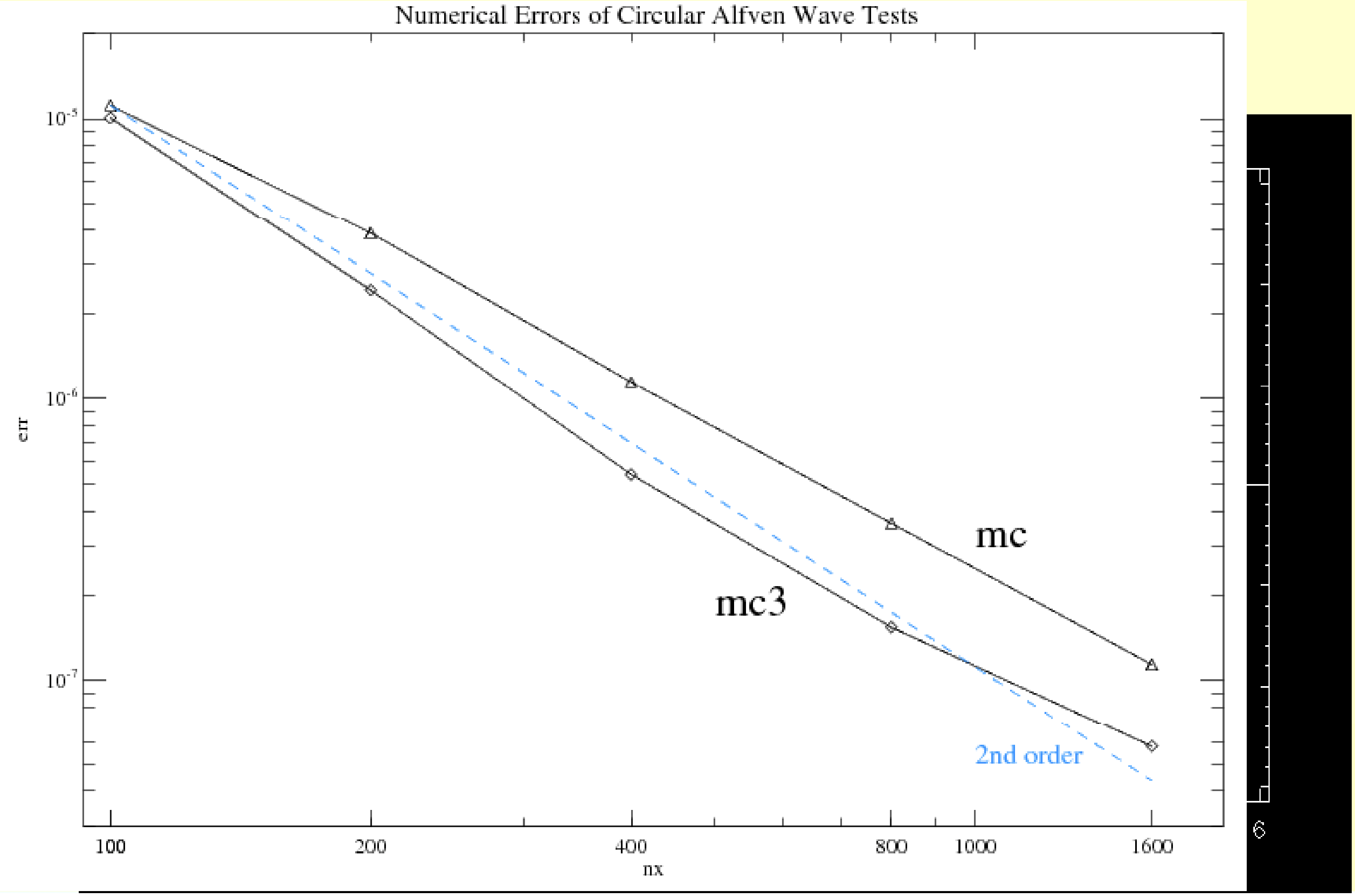
Sound wave propagating parallel to magnetic field at $c_s = \sqrt{3p_\parallel/\rho}$

Grid convergence for smooth problem:



Tests for anisotropic pressure

Circularly polarized Alfven wave propagating at $v_A = \sqrt{(\mathbf{B}^2 + p_\perp - p_{||})/\rho}$



Multi-Ion MHD

For each ion fluid *s* (neglecting resistive terms):

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s) = S_{\rho_s}$$

$$\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s + I p_s) = \frac{n_s q_s}{n_e e}(\mathbf{J} \times \mathbf{B} - \nabla p_e) + n_s q_s (\mathbf{u}_s - \mathbf{u}_+) \times \mathbf{B} + S_{\rho_s \mathbf{u}_s}$$

$$\frac{\partial p_s}{\partial t} + \nabla \cdot (p_s \mathbf{u}_s) = -(\gamma - 1) p_s \nabla \cdot \mathbf{u}_s + S_{p_s}$$

Induction equation (neglecting Hall term):

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u}_+ \times \mathbf{B}) = 0$$

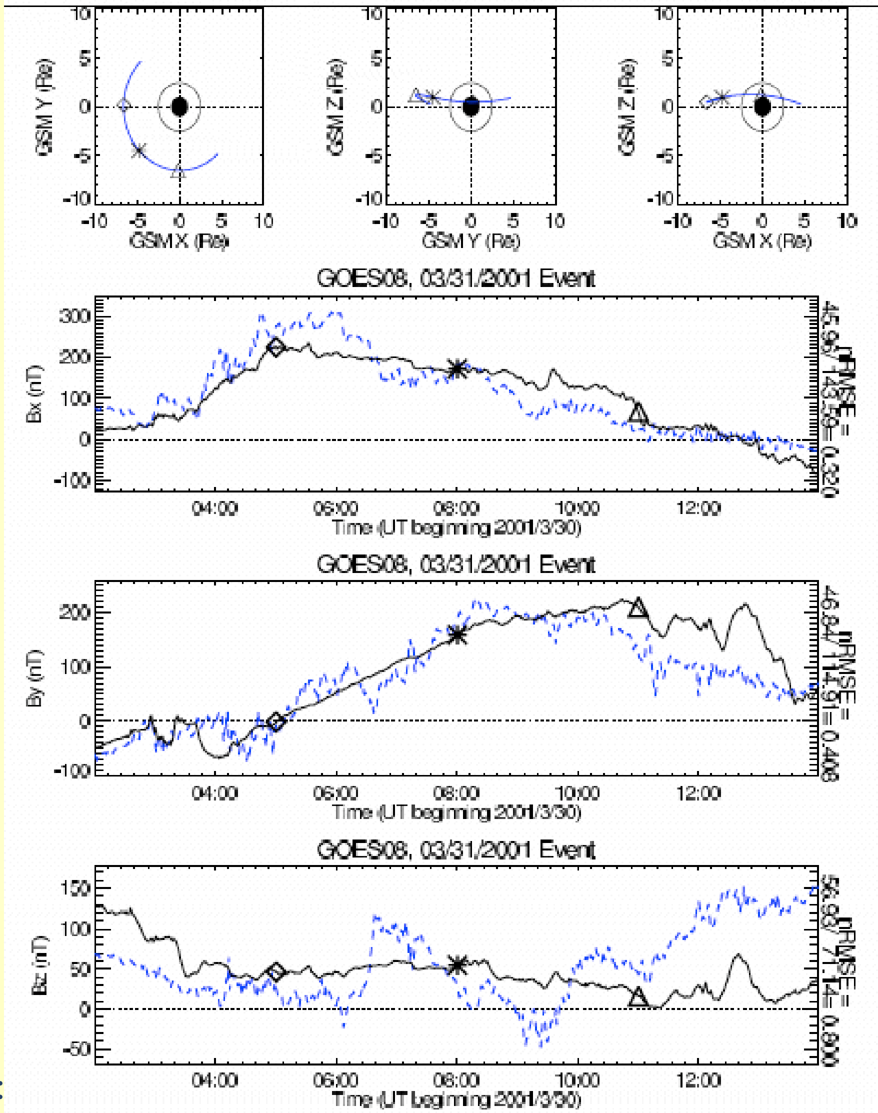
where the charge-averaged ion-velocity is $\mathbf{u}_+ = \frac{\sum_s n_s q_s \mathbf{u}_s}{e n_e}$

Initial Results with Multi-Ion MHD
(Glocer et al, in press)

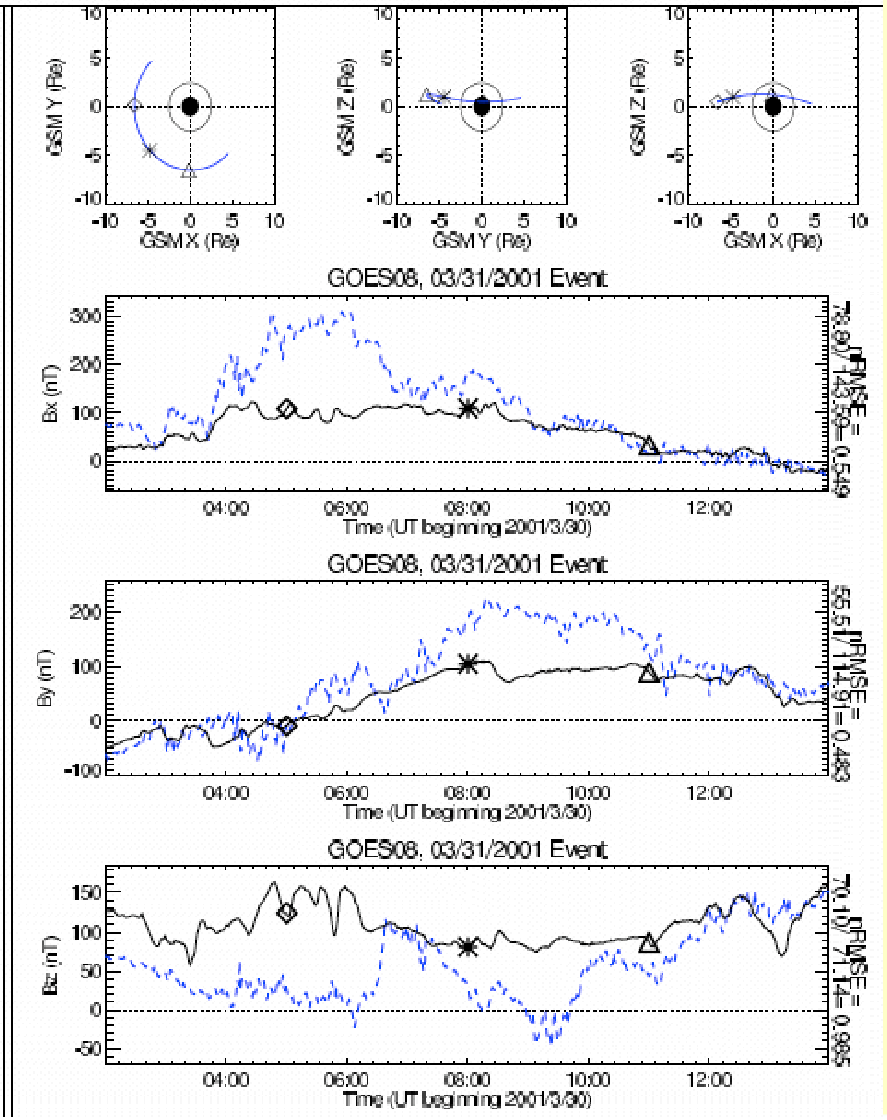
- **Modeling two magnetic storms**
  - May 4, 1998
  - March 31, 2001
- **Multi-fluid BATS-R-US running in the SWMF coupled with**
  - Polar Wind Outflow Model
  - Ridley Ionosphere-electrodynamics Model
  - Rice Convection Model (inner magnetosphere)
- **Comparison with**
  - single fluid model
  - global indexes (Dst, CPCP)
  - in situ satellite measurements

Magnetic Field vs Goes 8 Satellite

Multi-fluid MHD with O^+ outflow



Single-fluid MHD with no outflow



Local Time Stepping

- **Time stepping schemes**
  - Explicit scheme with fixed time step is limited by most restrictive CFL condition
  - Time step in an implicit scheme is not restricted by stability, but expensive
  - Local time stepping allows each cell/block advance in time limited by local CFL
- **Issues in Local Time Stepping algorithm**
  - Time interpolation (and extrapolation!) of ghost cells
  - Order of advancing the cells
  - Load balancing
  - Conservative flux correction

Local Time Stepping

- **Choices in Local Time Stepping algorithm**
  - Local time step is proportional to grid cell size (Berger and Colella)
    - + relatively simple
    - - not optimal if the fastest wave speed varies a lot in the computational domain
  - Local time step set by CFL condition and rate of change (Omelchenko et al)
    - + optimal time step
    - + stability is guaranteed
    - - complicated, difficult scheduling and load balancing
    - - multiple flux calculations between cells with similar time steps
  - Local time step set by CFL condition, but rounded to powers of 2 fractions (Flaherty et al., Crossley and Wright)
    - + close to optimal time step (within factor of 2)
    - + simpler scheduling and load balancing
    - + few extra flux calculations
    - - have to be careful about stability

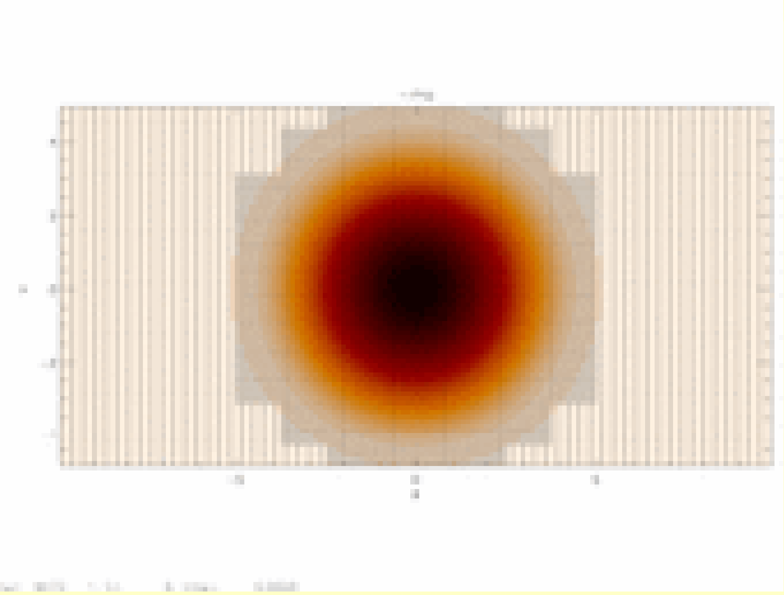
Local Time Stepping

Current status

- Local time step is set on a block by block basis based on grid level
- Each level is load balanced separately

Plan

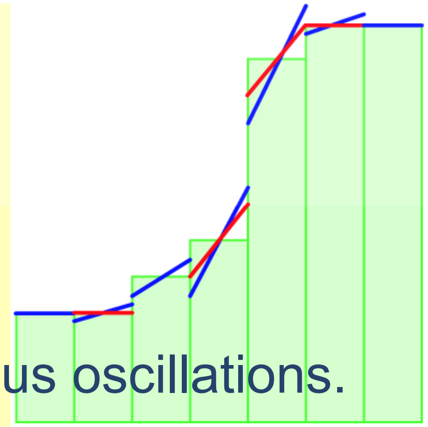
- Eventually use powers of 2 time steps based on local block CFL condition
- Possibly check stability condition during global time step and change local time step if needed, but no load balancing at this stage



Summary

- Added two-fluid (electron + ion) and anisotropic MHD to BATS-R-US.
- Initial verification tests pass for anisotropic pressure MHD.
- First multi-ion MHD results are now published (Glocer et al, JGR).
- Prototype local time stepping scheme has been implemented.

Numerical Diffusion



- **Numerical diffusion is useful**
  - Explicit scheme: needed for stability and to avoid spurious oscillations.
  - Implicit scheme: needed to avoid spurious oscillations.
  - High order schemes have less diffusion in well resolved regions only.
- **Total Variation Diminishing (TVD) schemes**
  - $F_{diff} = |\lambda| (U_R - U_L)/2$
  - Here U_R and U_L are the right and left states and λ is the wave speed, that is the Alfven speed for the magnetic field.
  - U_R-U_L is proportional to $(\Delta x)^n$ where n is the order of the scheme.
  - Near fast changing gradients and at local extrema TVD schemes become first order accurate: $U_R - U_L \approx \Delta x\, dU/dx$
- **Typical affordable grid resolution near Earth is 1/4 R_E to 1/8 R_E**
  - Far from well resolved, so first order numerical diffusion dominates.

Boris Correction

Semi-relativistic correction

- Classical Alfven speed $v_A = B/\sqrt{\mu_0 \rho}$ can exceed speed of light c.
- Boris (also Gombosi et al.) derived the semi-relativistic MHD equations, where

$$v_A^{semi} = \frac{v_A}{\sqrt{1 + v_A^2/c^2}}$$

- The semi-relativistic formulation is necessary for Saturn or Jupiter.

Reducing the speed of light

- One can replace c with a reduced c', e.g. c' = 0.01 c = 3000km/s
- + Allows larger time steps for explicit time stepping
- + Reduces the numerical diffusion by reducing $\lambda \approx v_A$!
- - Changes the time accurate behavior of the MHD equations…
- Artificial reduction of c is used in many magnetosphere codes

Limited Numerical Diffusion

- **New idea: limit diffusive flux in the implicit TVD scheme**
  - $F_{diff} = |\lambda'| (U_R - U_L)/2$ and $|\lambda'| = \min(\lambda_{max}, |\lambda|)$
  - λ_{max} can be set to any value, e.g. 2000km/s for magnetosphere simulation
  - + Numerical diffusion is reduced
  - + The physical behavior of the MHD equations is not modified
  - – Oscillation-free property is not guaranteed. But in practice, it seems to work.

- **Test: May 1998 magnetic storm**
  - Both Boris correction and limited numerical diffusion results in more negative Dst.
  - Closer to observations.