# Modeling of Surface Acoustic Wave Strain Sensor

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## Introduction

NASA is investigating the use of Surface Acoustic Wave (SAW) technology for Integrated Vehicle Health Monitoring (IVHM) of aerospace structures. To facilitate rapid prototyping of passive SAW strain sensors for aerospace applications, SAW models have been developed. Three methods for modeling SAWs are compared; impulse response method, the conventional matrix approach, and a modified matrix approach. Results from the models are presented with measured data from devices.

The first order model is based upon the rmpulse response method [1, 2]. For more accurate results, a matrix based approach is used [3]. This approach has been modified to include finger reflections that occur when the finger metal is too thick [4]. This work supports milestone 1.1.1.8.

## Impulse Response Model

The impulse response method [1] was used as the baseline for modeling the SAW device. This method is valid only for transducers where at least one of the interdigitated transducer (IDTs) has a constant aperture [3]. It models the mechanical and electrical behavior, the frequency response, loss of the system, and admittance of SAW devices. This model assumes constant and equal spacing and finger widths. A simple circuit model (Fig. 1) is used to convey the basic elements. The figure shows the source voltage and both the source and load impedances which are not part of the model.

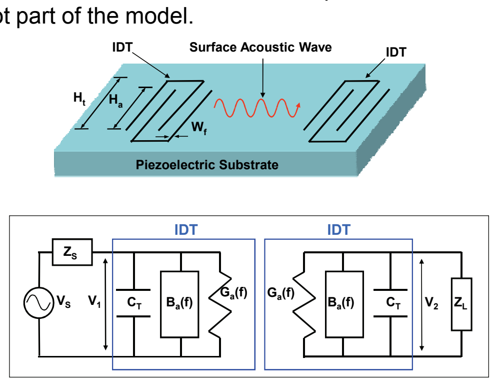


Fig. 1. (top) SAW delay line, (bottom) the impulse response circuit model $C_T$ is the total capacitance, $B_a(f)$ is the acoustic susceptance, and $G_a(f)$ is the radiation conductance.

The frequency response of a SAW device can be calculated by using the impulse response model and is given by

$$H(f) = 20\log\left(4k^2 C_s H_a f_0 N_p^2 \left(\frac{\sin\left(N_p \pi \frac{(f-f_0)}{f_0}\right)}{N_p \pi \frac{(f-f_0)}{f_0}}\right)^2 e^{-j\left(\frac{N_p + D}{f_0}\right)}\right)$$

where k is the piezoelectric coupling coefficient, $C_s$ is the capacitance per finger pair and unit length, $H_a$ is the aperture of the fingers, $f_0$ is the synchronous frequency, $N_p$ is the number of finger pairs, f is the frequency, and D is the delay length between the IDTs.

## Conventional Matrix Method

The matrix methods were developed using transmission line approach [3]. This method is based upon a circuit with both electrical and acoustic ports (Fig. 2). This allows the acoustic waves ($W_i$) and electrical parameters ($a_i$ and $b_i$) to be related through the use of transmission matrix T in:

$$\begin{pmatrix} W_{i-1}^{+} \\ W_{i-1}^{-} \\ b_i \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ -t_{12} & t_{22} & t_{23} \\ st_{13} & -st_{23} & t_{33} \end{pmatrix} \begin{pmatrix} W_i^{+} \\ W_i^{-} \\ a_i \end{pmatrix}.$$

Given the T matrix for an IDT, calculations for a SAW delay line can be performed. The matrix for a delay line is simply the multiplication of a 4x4 sub-matrix (elements t11, t12, t21, and t22) for the two IDTs and a matrix for the delay in between. The delay matrix is modeled on an acoustic transmission line, where λ is the wavelength (synchronous frequency) and d is the distance between the center of the two IDTs. Therefore complete SAW device matrix is given by:

$$[SAW(f)] = \left[T_1(f)\begin{bmatrix} e^{\frac{2\pi}{\lambda}d} & 0 \\ 0 & e^{\frac{-2\pi}{\lambda}d} \end{bmatrix} T_2(f)\right].$$

## Modified Matrix Method

The conventional matrix extensions add reflections from fingers [3, 4]. The transmission line matrix models un-metalized areas (1/8 λ) around one metal finger (1/4 λ). Each relates the voltages $V_1$ and $V_2$, to the currents $I_1$ to $I_2$ (Fig. 3). Where $Z_u$ and $Z_m$ are the acoustic impedances, $C_0$ is the capacitance of a finger, $\theta_u$ and $\theta_m$ are the acoustic angles of the substrate, and the transformer turns ratio is 1:1.

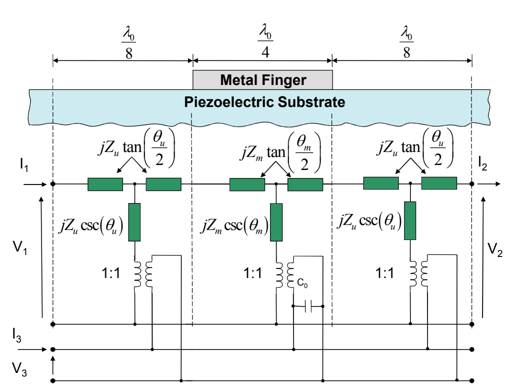


Fig. 3. Transmission line equivalent circuit for a single finger.

The transmission matrices for the metalized and un-metalized regions are given by:

$$[R_u(f)] = \begin{bmatrix} \cosh(j\theta_u(f)) & Z_u \sinh(j\theta_u(f)) \\ \frac{1}{Z_u}\sinh(j\theta_u(f)) & \cosh(j\theta_u(f)) \end{bmatrix} \qquad [R_m(f)] = \begin{bmatrix} \cosh(j\theta_m(f)) & Z_m \sinh(j\theta_m(f)) \\ \frac{1}{Z_m}\sinh(j\theta_m(f)) & \cosh(j\theta_m(f)) \end{bmatrix}$$

The IDT matrix is the combination of the finger matrices raised to the power of the number of electrodes. The SAW delay line matrix is the multiplication of the matrices for the two IDTs and the delay between the IDTs.

$$[IDT(f)] = [[R_u(f)][R_m(f)][R_u(f)]]^{2N_p}$$

$$[SAW(f)] = [IDT_1(f) D_1(f) IDT_2(f)]$$

## Results

All three methods capture the main lobe and two side lobes for 50 nm thick fingers (Fig. 4). But, the modified matrix captures the frequency shift due to the mass loading [5]. In Fig. 5, the first order model and conventional matrix results are both centered about the synchronous frequency. The measured results and the modified matrix results are both shifted in frequency due to velocity changes from mass loading effects. Comparing the measured data from the two figures, Fig. 4 does not have the same artifacts as Fig. 5. They are from bulk waves that reflect from the polished bottom surface. A rough non-polished surface disperses the bulk waves and reduces these artifacts. While the models work well for devices at 46.4 MHz, as the frequency increases the accuracy goes down for the side lobes (Fig. 6).

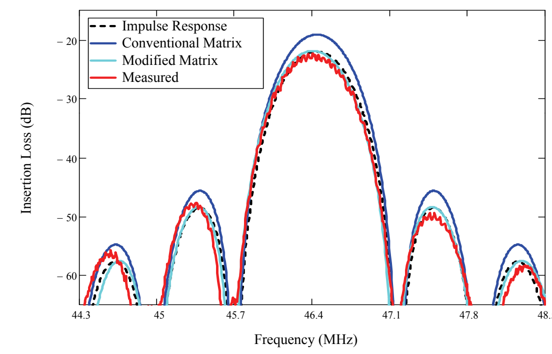


Fig. 4. Comparison of model results with data from a single side polish wafer, with 50 nm of aluminum.

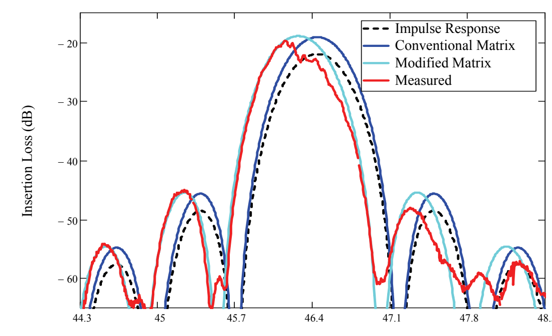


Fig. 5. Comparison of model results with data from a double side polish wafer, with 250 nm of aluminum.

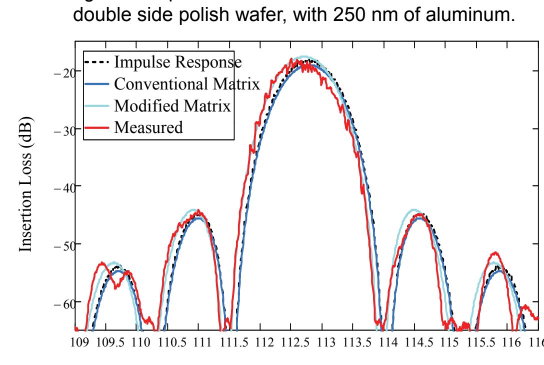


Fig. 6. Comparison of model results with data from a single side polish wafer, with 50 nm of aluminum.

## Conclusions

SAW prototype results are compared to the impulse response, a conventional matrix, and a modified matrix model. Ignoring mass loading from the fingers, all three are fairly accurate. However, the modified matrix method more accurately models second order effects such as frequency shift from mass loading. Neither the impulse response nor the conventional matrix method model the frequency shift. Therefore, the modified matrix method is the most accurate for a wider range of parameters such as finger metal thicknesses. These results will be used to develop the design of a SAW strain sensor.

## References


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[4] H. Y. Tung, "The Design of Surface Acoustic Devices using the Transmission Matrix Method," PhD Dissertation, I-Shou University, July, 2005.

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