# How Well Do You Know That?

## Uncertainty Analysis in Earth Remote Sensing

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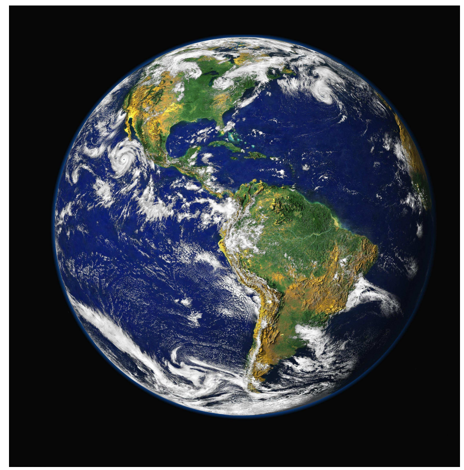
Supported by NASA Grant Numbers NNG06GI77G and NNX07AV69G

# Modeling The Biosphere Has Important Scientific and Public Policy Implications



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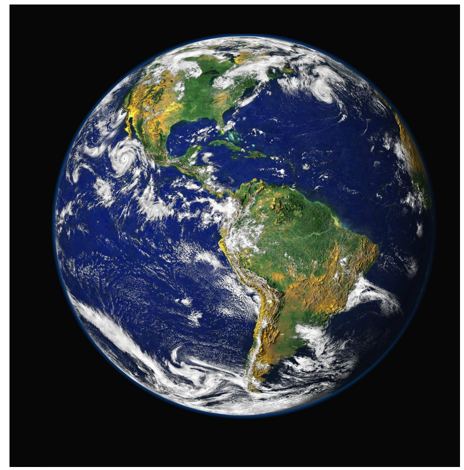

Forests are a major source for the transfer of mass and energy from land to the atmosphere

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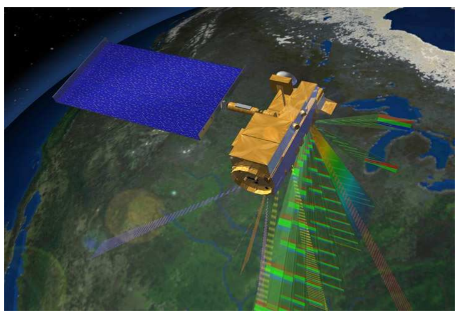

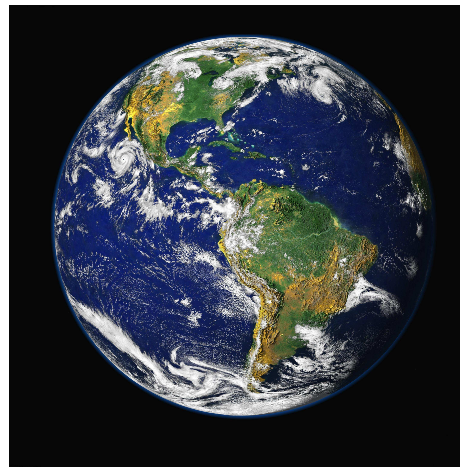

Also, you can see them from space!

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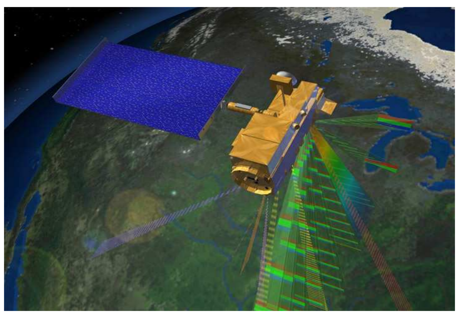
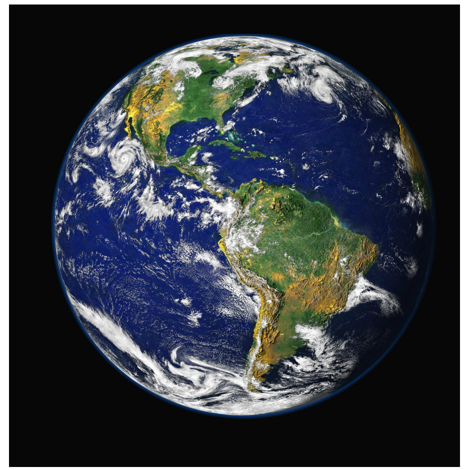
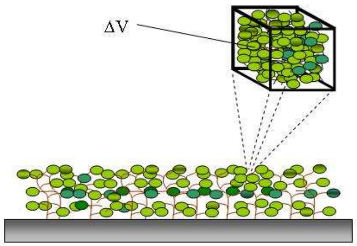




How well can we estimate forest parameters from remote-sensed data?

# Statistical Inference Provides a Methodology to Analyse Models and Provide Well-Calibrated Estimates




Analyse a computer model of light interaction with forest canopies.

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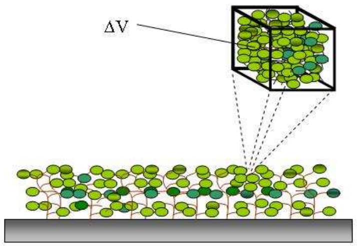


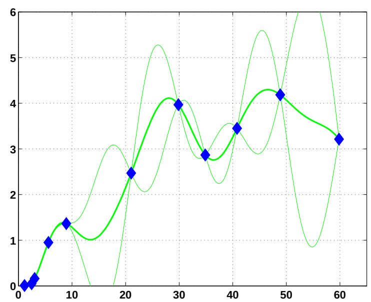


Emulate the computer model using a Gaussian Process.

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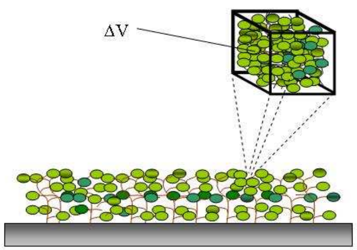
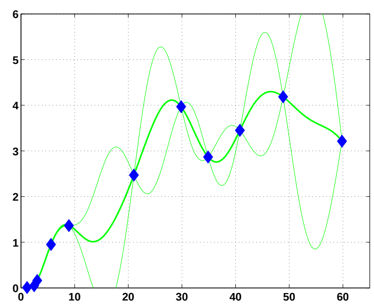
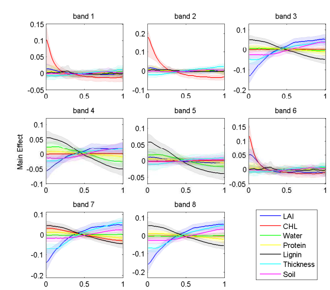
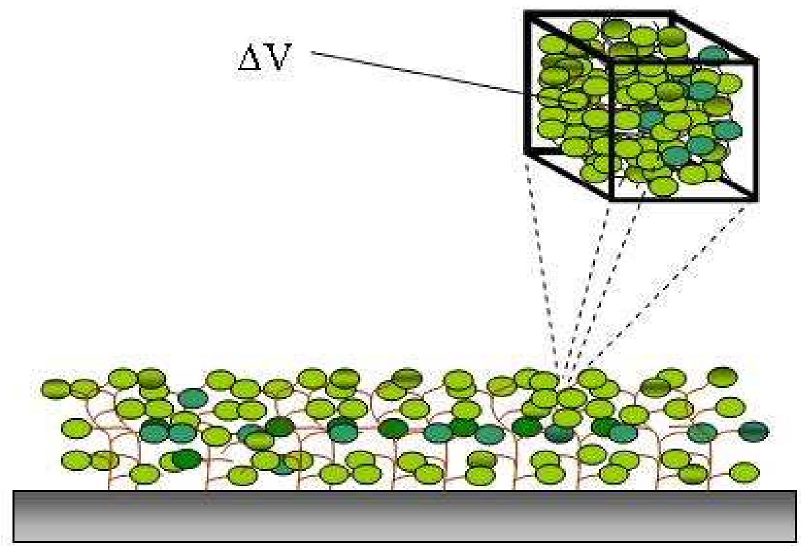



The Gaussian Process model is amenable to analysis.

# Radiative Transfer Models Simulate the Interaction of Light With Forest Canopies




CANopy MODel

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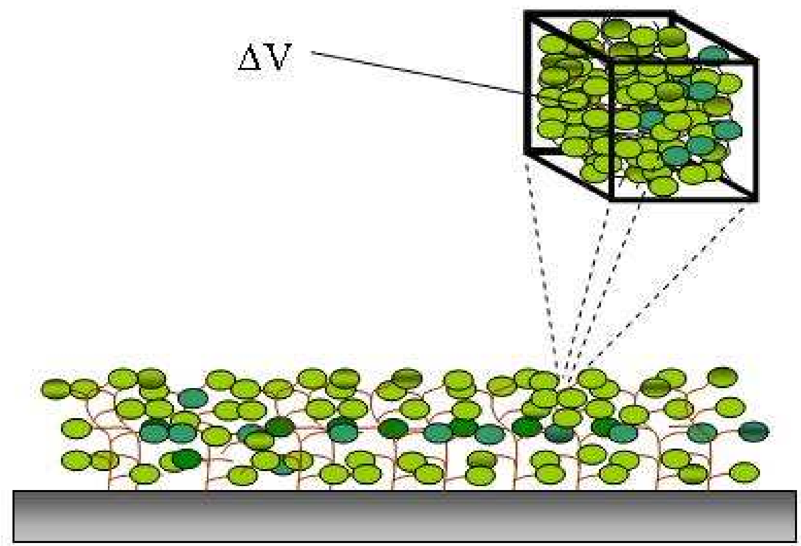


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Leaf Area Index (LAI)
Leaf Angle Distribution
Soil Reflectance

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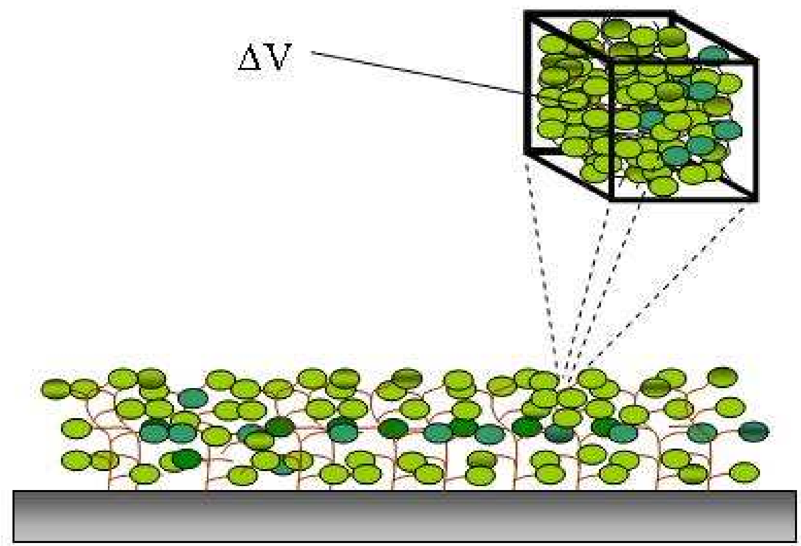


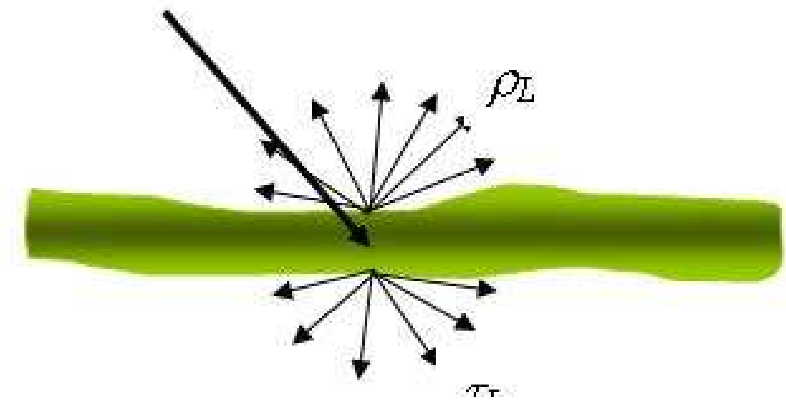


LEAF MODel

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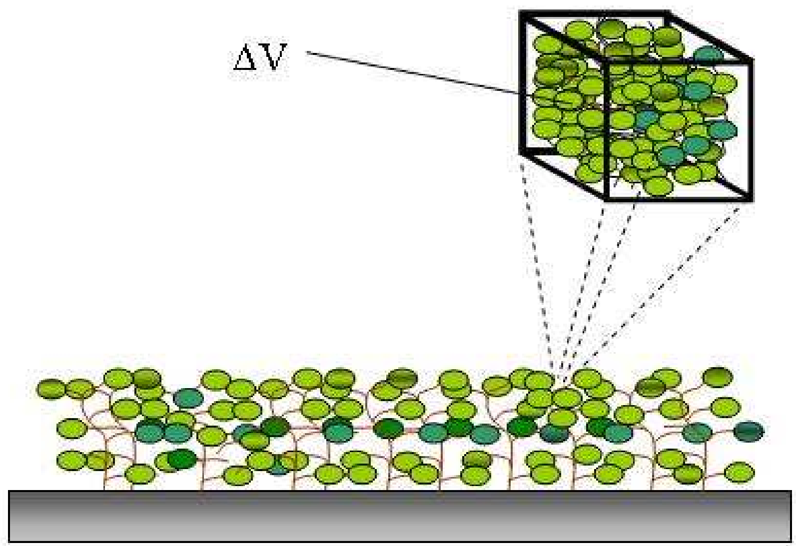


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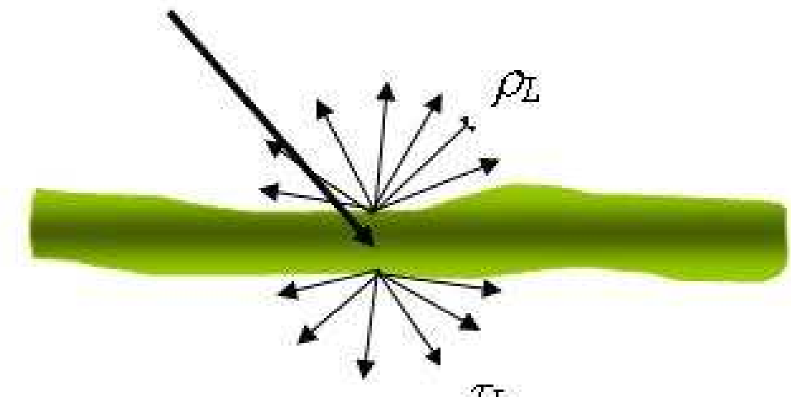

LEAF MODel

Chlorophyll
Water Fraction
Protein
Lignin/Cellulose
Thickness

# Global Sensitivity Analysis Reveals Important Characteristics of the Computer Model

Decompose the output of the LCM as

$$\begin{aligned} y = f(\boldsymbol{v}) &= \mathsf{E}(Y) + \sum_{i=1}^{d} z_i(v_i) + \sum_{i<j} z_{i,j}(v_i, v_j) + \dots \\ &\quad + z_{1,2,\dots,d}(v_1, v_2, \dots, v_d) \end{aligned}$$

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Global Mean $$\mathsf{E}(Y) = \int_{v_j, j=1\ldots d} f(\boldsymbol{v}) \mathsf{d}H(\boldsymbol{v})$$

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Global Mean

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Main Effects

$$\begin{aligned} z_i(v_i) &= \mathsf{E}(Y|v_i) - \mathsf{E}(Y) \\ &= \int_{\boldsymbol{v}_{-i}} f(\boldsymbol{v}) \mathsf{d}H(\boldsymbol{v}_{-i}|v_i) - \mathsf{E}(Y) \end{aligned}$$

## Sensitivity Indices - The Expected Reduction in Output Uncertainty if an Input Was Known Exactly

Variances of the components of the output decomposition

$$V_i = \mathsf{Var}\{\mathsf{E}(Y \mid v_i)\} = \mathsf{E}\left[(\mathsf{E}(Y \mid v_i))^2\right] - (\mathsf{E}(Y))^2.$$

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Normalize to give the Sensitivity Indices

$$S_i = \frac{V_i}{\mathsf{Var}(Y)}$$

How much of the variance of the output is due to input $i$.

If I learn the value of input $i$ exactly, by how much is the variance of the output reduced?

# There are Two Strategies for Computing the Integrals Involved in the Main Effects and Sensitivity Indices

Monte Carlo approximation of the analytic integral.

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Trade-off depends on the dimensionality.

# Use a Gaussian Process Emulator in Place of the LCM

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Specified by the

mean function $\mathsf{E}(f(\boldsymbol{v})) = \mu$

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Joint distribution of any finite set of points is multivariate Gaussian.

Parameters $(\mu, \sigma, \phi)$ learned using maximum likelihood from a set of training runs of the LCM.

# Taking Into Account the Uncertainty Introduced by the GP Approximation

Recall Main Effects $z_i(v_i) = \mathsf{E}(Y|v_i) - \mathsf{E}(Y)$

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$$\mathsf{E}^* \left\{ \mathsf{E}(Y) \right\} = \hat{\mu} + \boldsymbol{T}^T C^{-1} (\boldsymbol{y} - \hat{\mu} \mathbf{1}_n)$$

$\mathsf{E}^* \{\}$ is expectation wrt GP
$\boldsymbol{T}$ – $n \times 1$ vector with elements
$\prod_{\ell=1}^{d} \left\{ \int_{a_\ell}^{b_\ell} \exp(-\phi_l |v_\ell - x_{i\ell}|^\alpha)(b_\ell - a_\ell)^{-1} dv_\ell \right\}$

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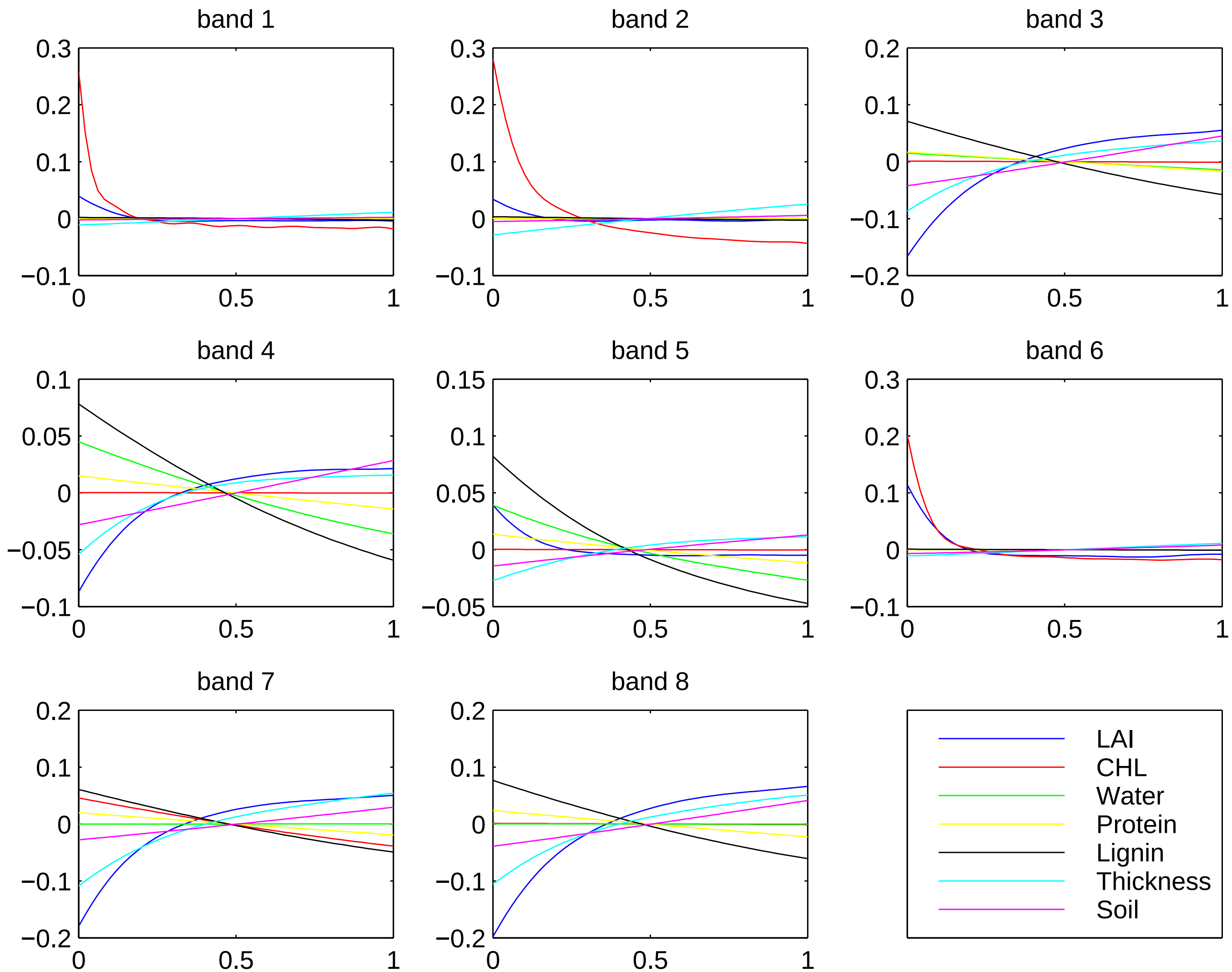
$\mathsf{E}^*\left\{\mathsf{E}(Y \mid u_j)\right\}$ can be similarly derived

This gives point estimates for the main effects.

# Main Effects for the Leaf-Canopy Model



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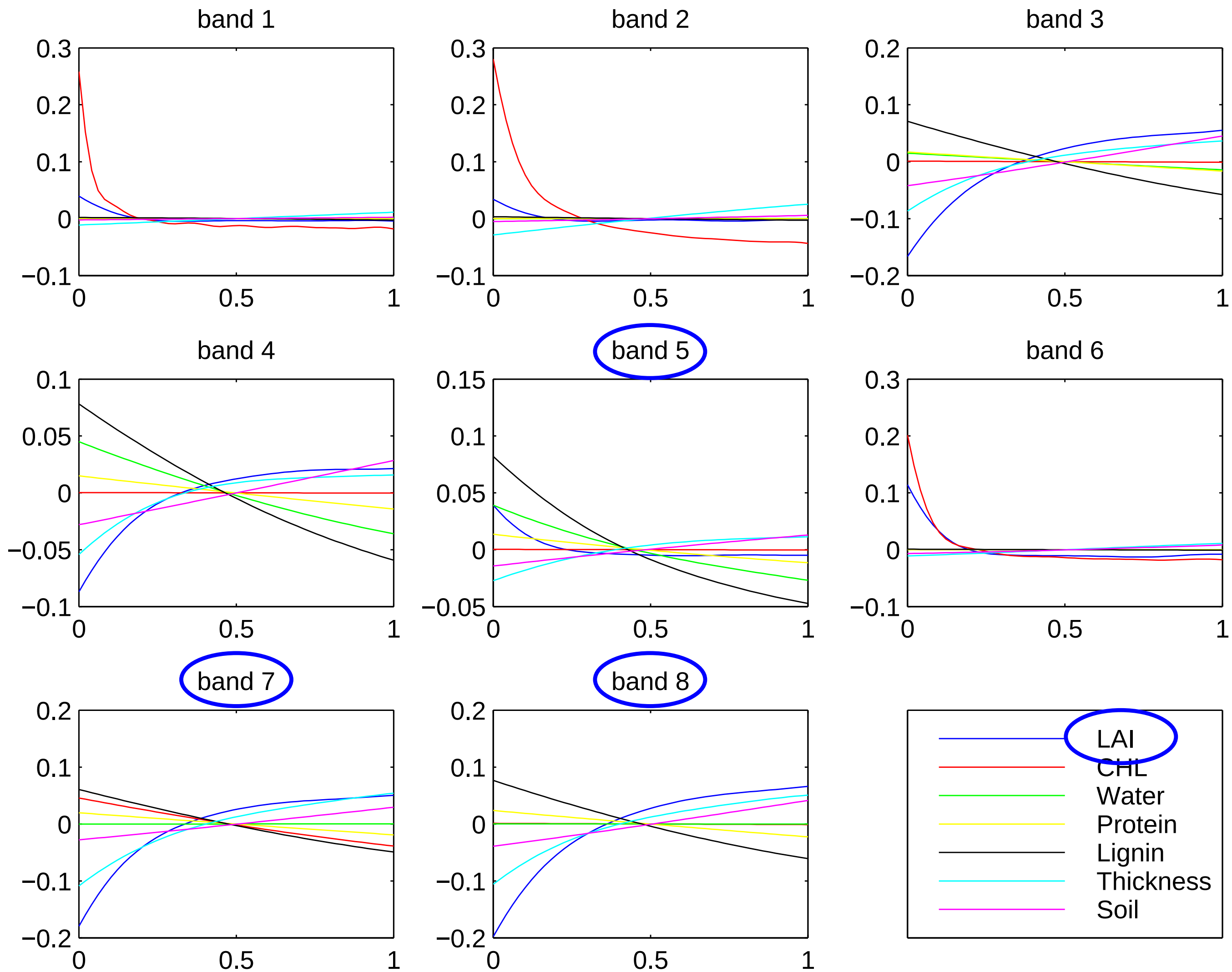


LAI: affects NIR (bands 7,8); opposite effect in visible (band 5)

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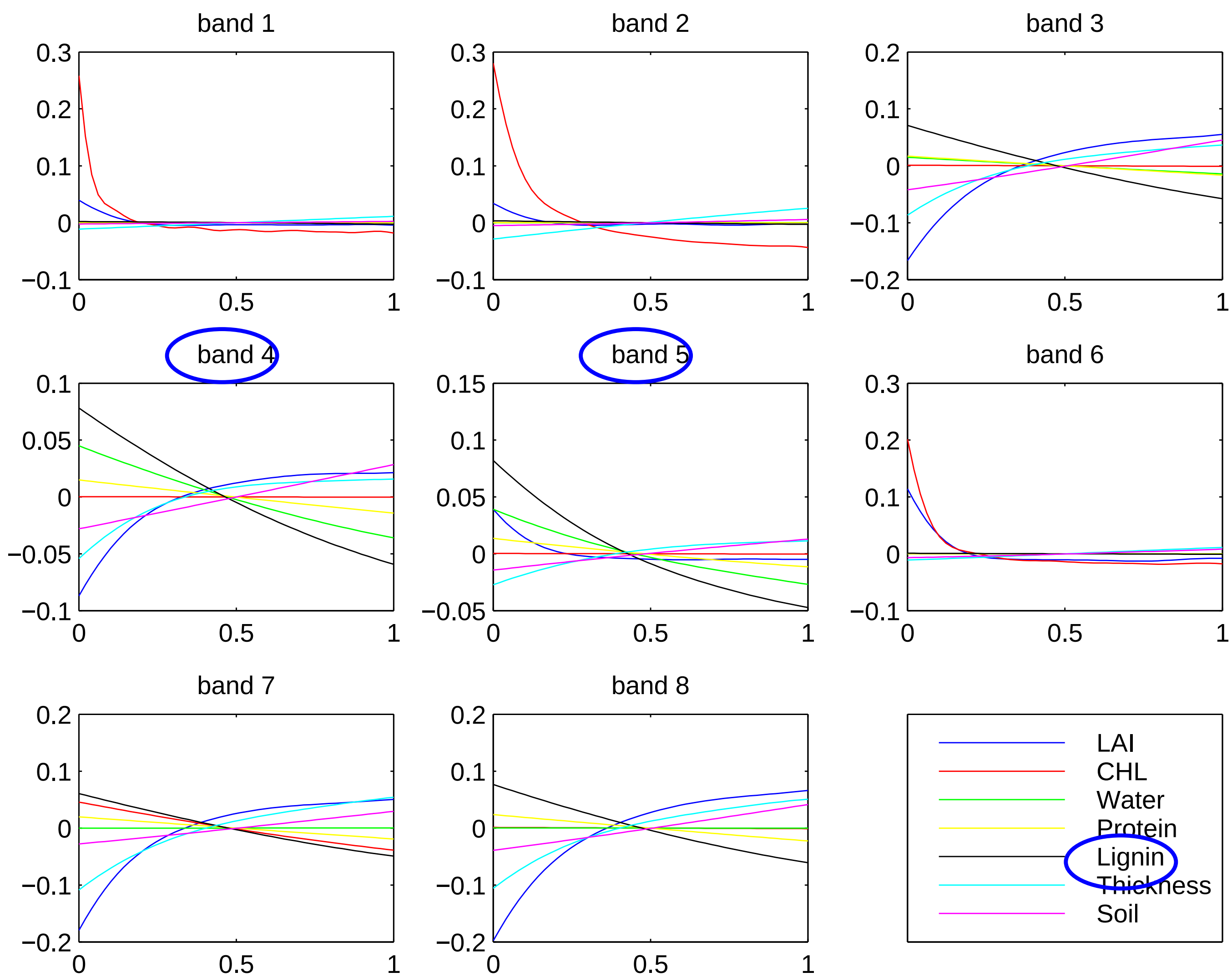


Lignin: affects SWIR (bands 4,5); surprising to domain scientists

# Estimating the Uncertainty of the Main Effects

Take $\text{Var}^* \{\}$ – variances wrt GP predictive distribution as a measure of this uncertainty

$$\text{Var}^* \{\text{E}(Y \mid u_j)\} = \text{E}^* \left\{(\text{E}(Y \mid u_j))^2\right\} - (\text{E}^* \{\text{E}(Y \mid u_j)\})^2.$$

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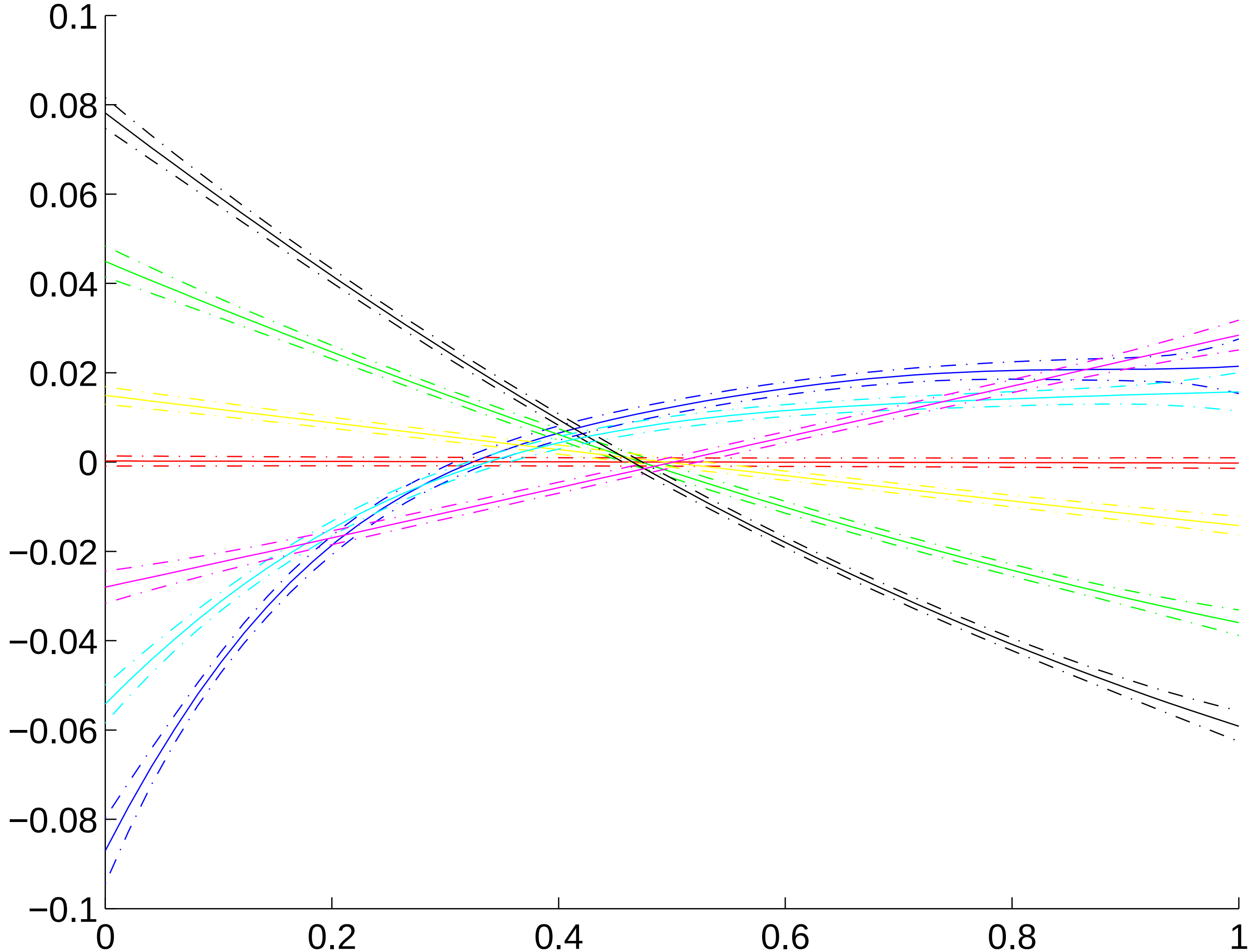
For our modeling choices (constant mean; exponential or squared exponential correlation function; uniform priors on the inputs) this can be computed analytically.

# Main Effects With Uncertainties




The uncertainties due to the GP approximation are small.

# Sensitivity Indices Under the GP Approximation

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Approximate by

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# Sensitivity Indices for the Leaf-Canopy Model

| | band; wavelength ($nm$) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| input | 469 | 555 | 1240 | 1640 | 2130 | 667 | 748 | 870 |
| LAI | 0.05 | 0.01 | 0.43 | 0.16 | 0.04 | 0.28 | 0.41 | 0.48 |
| CHL | 0.80 | 0.83 | 0.00 | 0.00 | 0.00 | 0.56 | 0.08 | 0.00 |
| Water | 0.00 | 0.00 | 0.01 | 0.12 | 0.14 | 0.00 | 0.00 | 0.00 |
| Protein | 0.00 | 0.00 | 0.01 | 0.02 | 0.02 | 0.00 | 0.02 | 0.02 |
| Lignin | 0.00 | 0.00 | 0.19 | 0.36 | 0.53 | 0.00 | 0.13 | 0.16 |
| Thick. | 0.02 | 0.05 | 0.14 | 0.07 | 0.05 | 0.02 | 0.24 | 0.18 |
| Soil | 0.00 | 0.00 | 0.08 | 0.06 | 0.03 | 0.01 | 0.03 | 0.06 |
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The GP parameters are estimated from a 250 point Latin Hypercube sampling of the input space.

There is significant uncertainty in the estimated GP parameters.

# Bayesian Estimation using MCMC to Include GP Parameter Uncertainty

Generate samples of the GP parameters

$$\boldsymbol{\psi} = (\boldsymbol{\theta}, \mu, \sigma^2, \boldsymbol{\phi})$$

where $\boldsymbol{\theta}$ are the predicted outputs at the training points.

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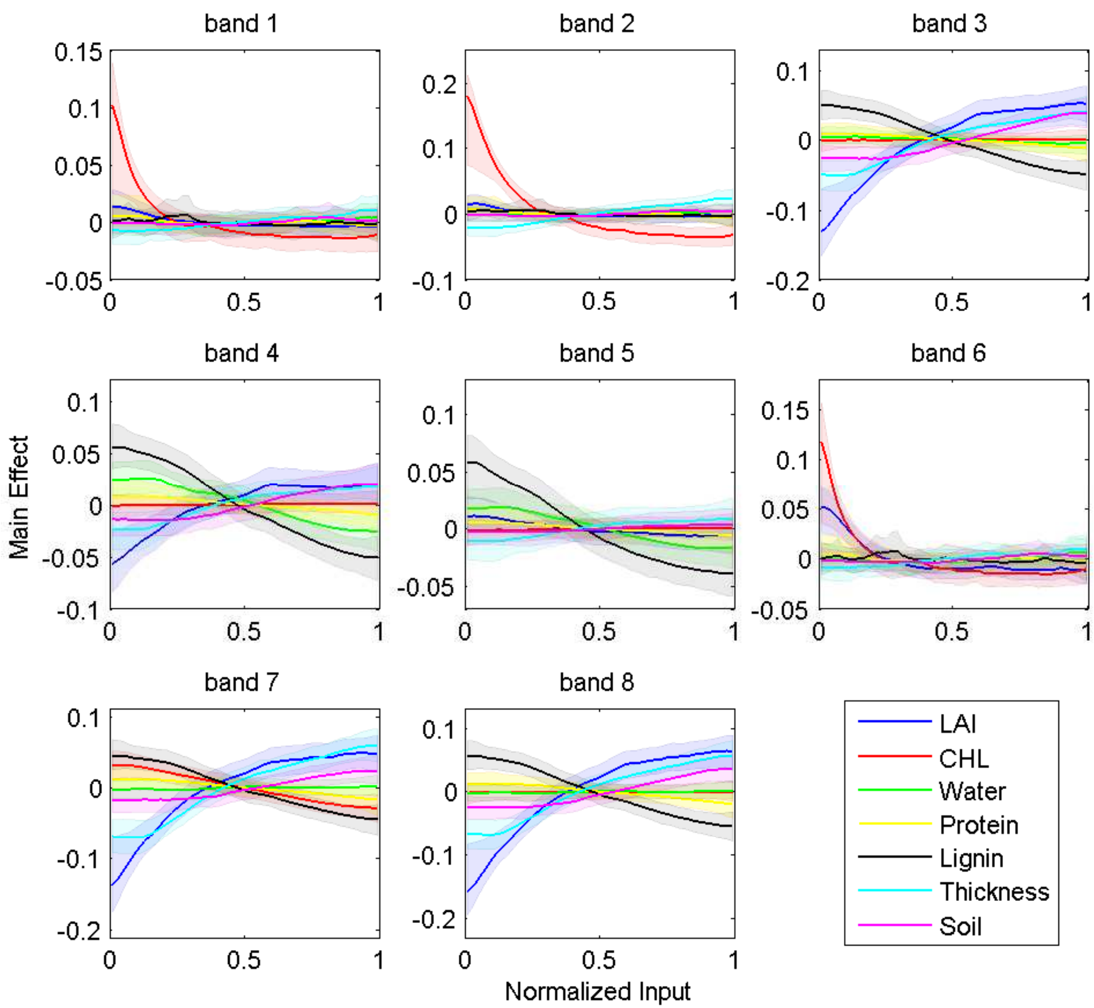
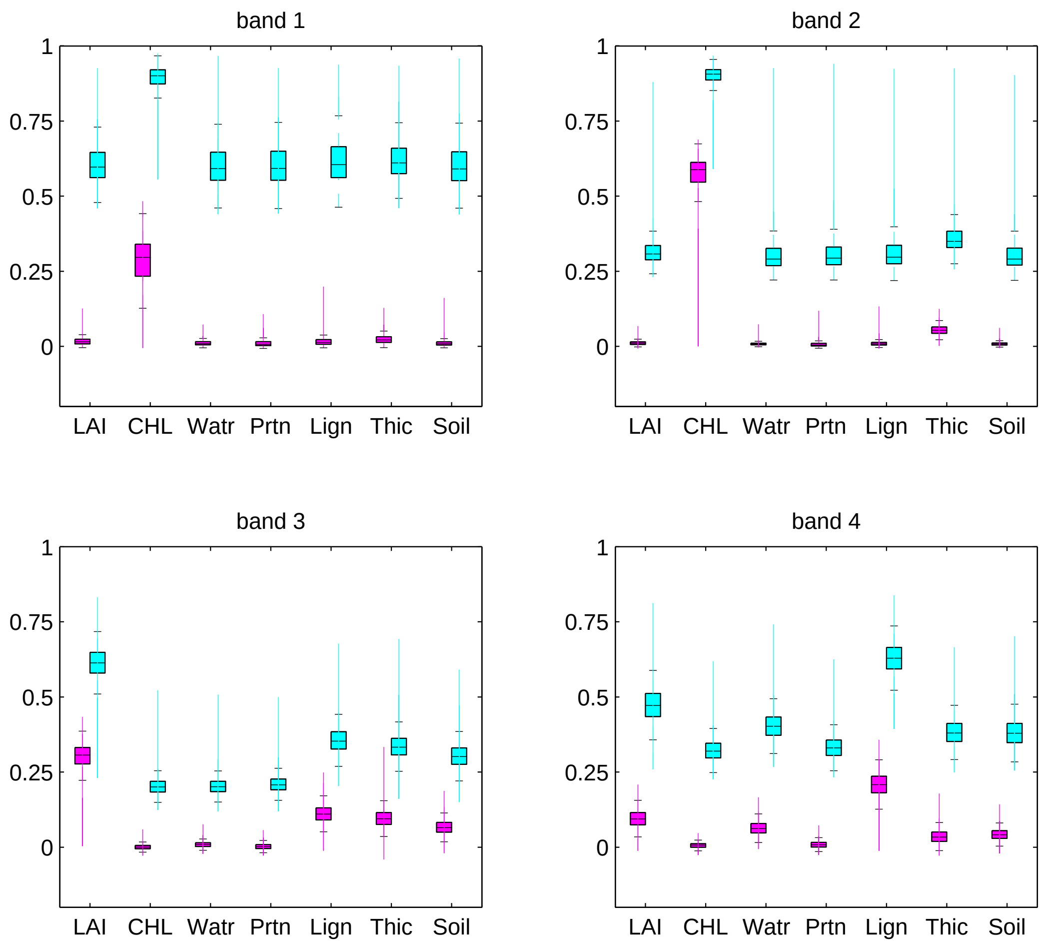
Can use these to estimate the full distribution of the main effects and the sensitivity indices.

# Median and 95% Probability Bands of the Posterior Distributions of the Main Effects




Uncertainties are larger, especially at the extreme values of the inputs. Basic behaviour is the same.

# Distributions of the Sensitivity Indices

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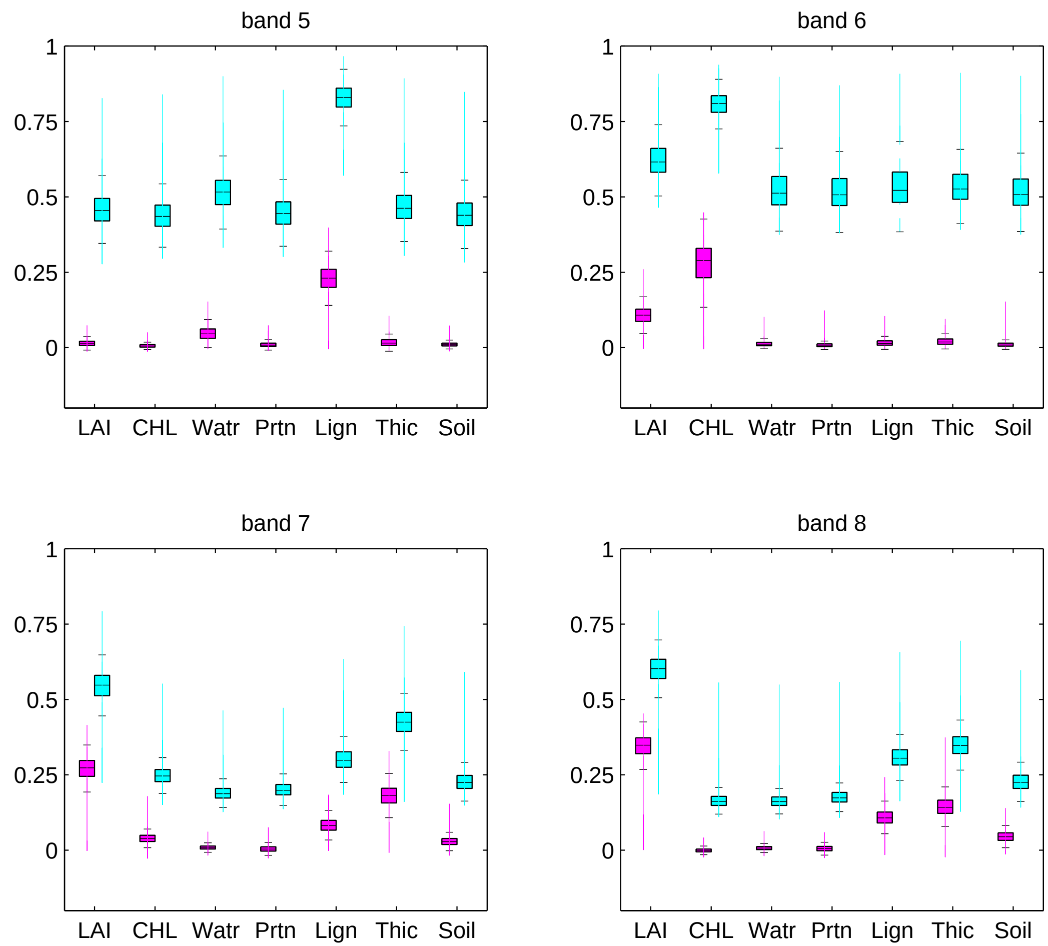


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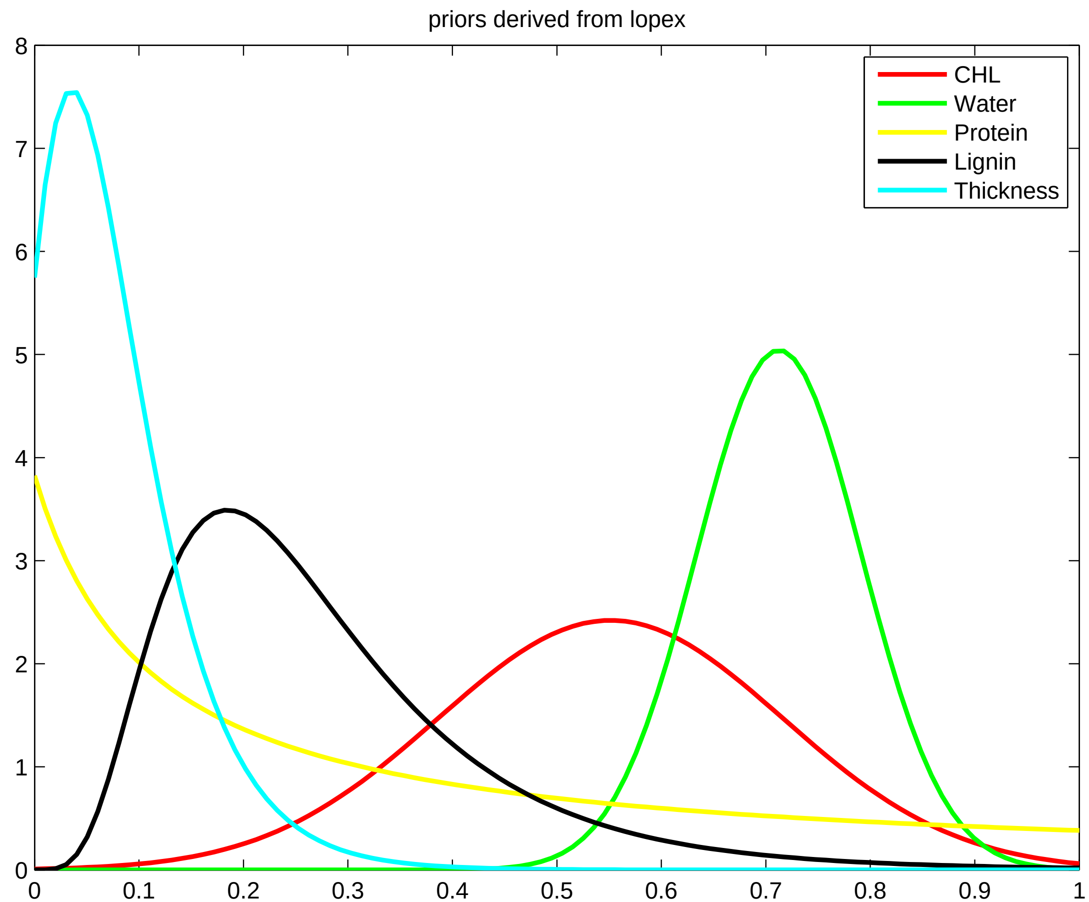
# Reducing the Uncertainty on the Inputs

Better priors on the inputs from analysis of the LOPEX (Leaf Optical Properties EXperiment) database.



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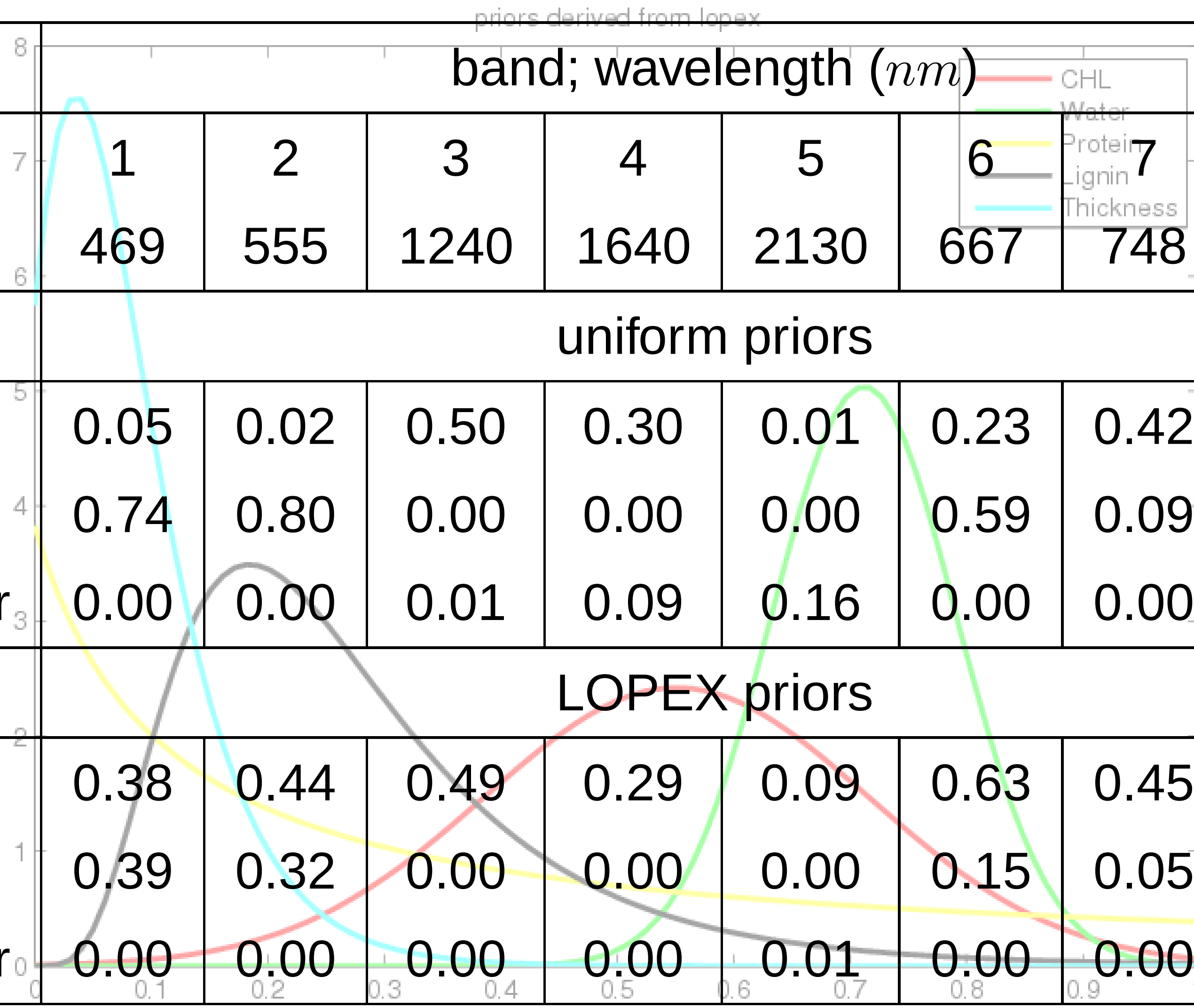


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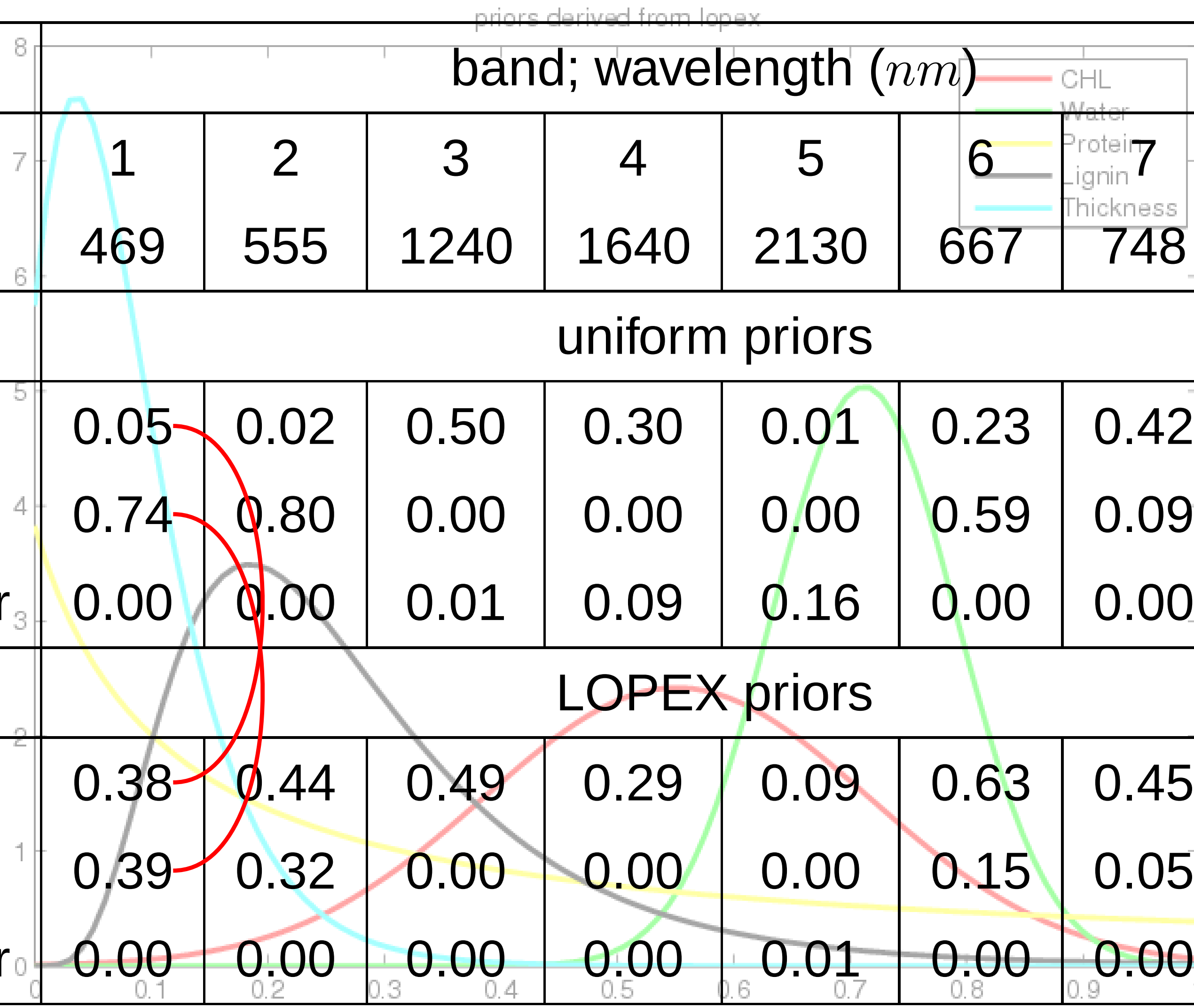
| LAI | 0.05 | 0.02 | 0.50 | 0.30 | 0.01 | 0.23 | 0.42 | 0.53 |
| CHL | 0.74 | 0.80 | 0.00 | 0.00 | 0.00 | 0.59 | 0.09 | 0.00 |
| Water | 0.00 | 0.00 | 0.01 | 0.09 | 0.16 | 0.00 | 0.00 | 0.00 |
| | LOPEX priors | | | | | | | |
| LAI | 0.38 | 0.44 | 0.49 | 0.29 | 0.09 | 0.63 | 0.45 | 0.55 |
| CHL | 0.39 | 0.32 | 0.00 | 0.00 | 0.00 | 0.15 | 0.05 | 0.00 |
| Water | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 |

# Reducing the Uncertainty on the Inputs




| | band; wavelength ($nm$) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| input | 469 | 555 | 1240 | 1640 | 2130 | 667 | 748 | 870 |
| | uniform priors | | | | | | | |
| LAI | 0.05 | 0.02 | 0.50 | 0.30 | 0.01 | 0.23 | 0.42 | 0.53 |
| CHL | 0.74 | 0.80 | 0.00 | 0.00 | 0.00 | 0.59 | 0.09 | 0.00 |
| Water | 0.00 | 0.00 | 0.01 | 0.09 | 0.16 | 0.00 | 0.00 | 0.00 |
| | LOPEX priors | | | | | | | |
| LAI | 0.38 | 0.44 | 0.49 | 0.29 | 0.09 | 0.63 | 0.45 | 0.55 |
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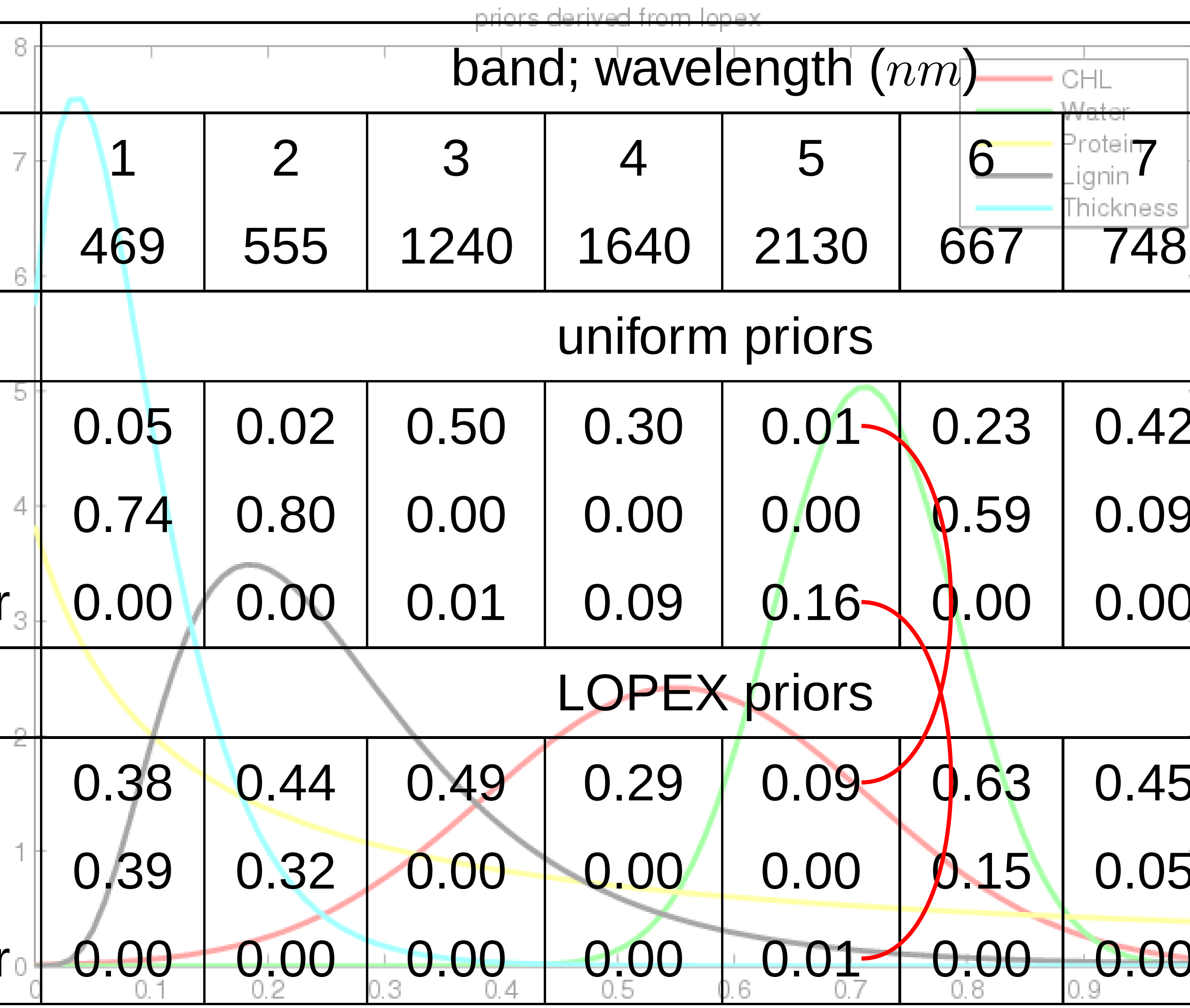


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Calibrating the LCM by estimating a bias function from areas where there are both field data and remote sensed data.

# Conclusions

Developed main effects and sensitivity indices for the LCM RTM

Extended the framework to account for uncertainty in the estimated GP emulator.

Provides insight for model improvement.

Results provided new information to the domain scientists.

Extending this work to validation and inversion.

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Questions?